MATEMATIKA: 13/14

1) $f(x) = \ln(x+1) - \frac{1}{2}x^2$

a) rovnice tečny v bodě $[x_0; f(x_0)]$; $x_0 = 0$

$\rightarrow y = \ln(0+1) - \frac{1}{2}\cdot 0^2$

$y = \ln(1) - 0$

$y = 0 \quad \rightarrow \quad T = [x_0; y_0] = [0; 0]$

$f'(x) = \frac{1}{x+1}\cdot(1+0) - \frac{1}{2}\cdot 2x = \frac{1}{x+1} - x$ $\quad \cancel{f'(x_0)=}$

$f'(x_0) = 1\cdot 1 - \frac{1}{2}\cdot 2\cdot 0 = 1$

$\rightarrow$ derivace = směrnice tečny

$$\boxed{t:\ y = f'(x_0)\cdot(x-x_0) + y_0}$$

$t:\ y = 1\cdot(x-0) + 0 \rightarrow \underline{\underline{y = x}}$

b) Taylor $T_2(x)$ stupně 2, okolí $x_0 = 0$, přibliž. hodnota $f(x)$ pro $x = \frac{1}{3}$

$$\boxed{T_n(x) = f(x_0) + \frac{f'(x_0)}{1!}\cdot(x-x_0) + \frac{f^{(n)}(x_0)}{n!}\cdot(x-x_0)^n}$$

$T_2(x) = f(x_0) + \frac{f'(x_0)}{1}\cdot(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2$

nemám $f''(x_0) \Rightarrow$ $f'(x) = \frac{1}{x+1} - x = (x+1)^{-1} - x$

$f''(x) = -1\cdot(x+1)^{-2}\cdot 1 - 1 = -\frac{1}{(x+1)^2} - 1$

$f''(x_0) = -\frac{1}{(0+1)^2} - 1 = -1 - 1 = -2$

$T_2(x) = 0 + \frac{1}{1}\cdot(x-0) + \frac{-2}{2}\cdot(x-0)^2 = x - x^2$

$T_2\left(\frac{1}{3}\right) \doteq \frac{1}{3} - \left(\frac{1}{3}\right)^2 = \frac{1}{3} - \frac{1}{9} = \frac{3}{9} - \frac{1}{9} \doteq \underline{\underline{\frac{2}{9}}}$

c) Lagrangeův tvar zbytku $R_3(x)$

↳ odhad chyby $R_3(\frac{1}{3})$ ve tvaru zbytku

$$\boxed{R_{n+1}(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}\cdot(x-x_0)^{n+1}} \quad ; \xi \text{ leží mezi } x \text{ a } x_0$$

$$f'''(x) = \frac{2}{(x+1)^3} \Rightarrow R_3(x) = \frac{\frac{2}{(\xi+1)^3}}{6}\cdot(x-0)^3$$

$$R_3(x) = \frac{1}{3(\xi+1)^3}\cdot x^3 \qquad \xi\in(0;\tfrac{1}{3})$$

$$R_3(\tfrac{1}{3}) \leq \left|\frac{1}{3\cdot(0+1)^3}\cdot\left(\frac{1}{3}\right)^3\right| = \left|\frac{1}{3}\cdot\frac{1}{27}\right| = \left|\frac{1}{81}\right|$$

$$R_3(\tfrac{1}{3}) \leq \left|\frac{1}{81}\right|$$

② hom. dif. rce $\ddot{x}+\dot{x}-6x=0$

a) FS, OŘ ; max. řešení Cauchyho úlohy pro poč. podm. $x(0)=0$, $\dot{x}(0)=5$

$$\lambda^2+\lambda-6=0 \to \lambda_2=-3 \quad \lambda_1=2$$

$*\,(e^{\lambda_1 t}v_1,\ e^{\lambda_2 t}v_2)$

$(c_1e^{\lambda_1 t}v_1 + c_2e^{\lambda_2 t}v_2)$

FS: $\varphi_1(t)=e^{2t}$, $\varphi_2(t)=e^{-3t}$

FS: $\{\varphi_1(t)=e^{2t};\ \varphi_2(t)=e^{-3t}\}$

OŘ: $\boxed{x(t)=c_1e^{2t}+c_2e^{-3t}}$; $t\in(-\infty;\infty)$

Cauchy: $x(0)=0$; $\dot{x}(0)=5$ $\to$ $\dot{x}(t)=2c_1e^{2t}-3c_2e^{-3t}$

$$0=c_1e^{2\cdot0}+c_2e^{-3\cdot0} \to 0=c_1+c_2 \to c_1=-c_2$$

$$5=2\cdot c_1\cdot e^{2\cdot0}-3c_2e^{-3\cdot0} \to \frac{5=2c_1-3c_2}{5=2c_1+3c_1} \to c_1=1, \quad c_2=-1$$

$$\boxed{x(t)=e^{2t}-e^{-3t} \quad ; t\in(-\infty;\infty)}$$

b) metodou odhadu -> partikulární řešení nehomog. rce

$\ddot{x} + \dot{x} - 6x = 10e^{2t}$ -> OŘ

$f(t) = 10e^{2t}$ -> polynom 0. stupně

(α je u e a β je u $\cos$)

$\alpha = 2$; $\beta = 0$ -> $\omega = \alpha + \beta i = 2$ -> $\omega = \lambda_1 \neq \lambda_2$ -> $k = 1$

$$x_p = e^{\alpha t} \cdot (A_0 \cos \beta t + B_0 \sin \beta t) \cdot t^k$$

$x_p = e^{2t} \cdot (A_0 \cos 0 + B_0 \sin 0) \cdot t$

$x_p = e^{2t} \cdot A_0) \cdot t$ -> $A_0 \cdot e^{2t} t$

$\dot{x}_p = A_0 \cdot (e^{2t} \cdot 2 \cdot t + e^{2t} \cdot 1) = 2A_0 e^{2t} t + A_0 e^{2t}$

$\ddot{x}_p = 2A_0 \cdot (2e^{2t} t + e^{2t}) + 2A_0 e^{2t}$

$= 4A_0 e^{2t} t + 2A_0 e^{2t} + 2A_0 e^{2t}$

$= 4A_0 e^{2t} t + 4A_0 e^{2t}$

DOSADIT DO PŮV. RCE

$\cancel{4A_0 e^{2t} t} + 4A_0 e^{2t} + \cancel{2A_0 e^{2t} t} + A_0 e^{2t} - \cancel{6A_0 e^{2t} t} = 10e^{2t}$

$5A_0 e^{2t} = 10e^{2t}$

$A_0 e^{2t} = 2e^{2t}$

$A_0 = 2 \Rightarrow \underline{\underline{x_p = 2e^{2t} t}}$

OŘ nehom. rce: $\underline{\underline{x(t) = c_1 e^{2t} + c_2 e^{-3t} + 2e^{2t} t \; ; \; t \in (-\infty; \infty)}}$

(3) načrtněte plochu Q (část paraboloidu)

$$Q = \{[x,y,z] \in E_3 \, ; \, z = 4 - x^2 - y^2 \, ; \, z \geq 0\}$$

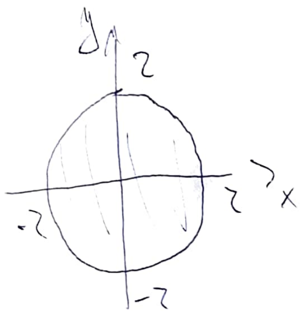
a) -> navrhněte parametrizaci a napište vektor kolmý k ploše Q

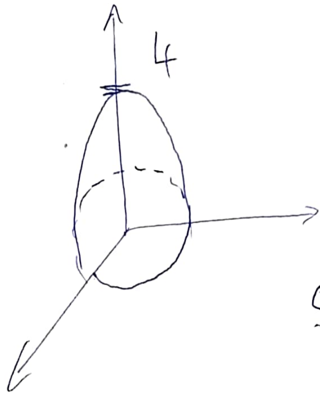
-> tok vekt. pole $\vec{f} = (x, y, 0)$ plochou Q

↳ $\iint_Q \vec{f} \cdot d\vec{p}$

NÁČRT

$\Rightarrow x^2 + y^2 \leq 4$





CILINDRICKÉ SOUŘADNICE:

$x = r \cdot \cos u$

$y = r \sin u$

$z = z$

$J = r$

PARAMETRIZACE:

$0 \leq r \leq 2 \; (r)$

$0 \leq u \leq 2\pi \; (\varphi)$

$x = r\cos(u)$

$y = r\sin(u)$

$\sin^2 u + \cos^2 u = 1$

$$z = 4 - x^2 - y^2 = 4 - (x^2 + y^2) = 4 - (r^2\cos^2(u) + r^2\sin^2(u))$$

$$= 4 - r^2 \cdot \underbrace{(\cos^2(u) + \sin^2(u))}_{1} = \underline{4 - r^2}$$

$$\boxed{\begin{aligned} p &= (r\cos(u);\, r\sin(u);\, 4 - r^2) \\ p_r &= \left(\frac{\partial x}{\partial r};\, \frac{\partial y}{\partial r};\, \frac{\partial z}{\partial r}\right) = (\cos(u);\, \sin(u);\, -2r) \\ p_u &= \left(\frac{\partial x}{\partial u};\, \frac{\partial y}{\partial u};\, \frac{\partial z}{\partial u}\right) = (-r\sin(u);\, r\cos(u);\, 0) \end{aligned}}$$

$$P_u \cdot P_r = \vec{n}_Q = \begin{vmatrix} i & j & k \\ -r\sin(u); & r\cos(u); & 0 \\ \cos(u) & \sin(u) & -2r \end{vmatrix} = (-2r^2\cos u; -2r^2\sin(u); -r)$$

$\sin^2 + \cos^2 = 1$

$\bullet\ n_Q \cdot (0;0;1) = -r \Rightarrow$ znaménko $\ominus$

kol. $\vec{f} = (x, y, 0)$

$$\iint_Q \vec{f}\, d\vec{p} = -\iint_Q \vec{f} \cdot \vec{n}_Q\, dp = -\int_0^2\int_0^{2\pi} (r\cos u; r\sin u; 0) \cdot (-2r^2\cos u; -2r^2\sin u; -r)\ du\,dr =$$

$$= -\int_0^2\int_0^{2\pi} -2r^3\cos^2 u - 2r^3\sin^2 u\ du\,dr = \int_0^2\int_0^{2\pi} +2r^3\ du\,dr =$$

$$= \int_0^2 2r^3 \cdot [u]_0^{2\pi}\, dr = 4\pi\int_0^2 r^3\, dr = \cancel{4}\pi \cdot \left[\frac{r^4}{\cancel{4}}\right]_0^2 = \underline{\underline{16\pi}}$$

b) kol. $\vec{f}$ plochou σ; VNĚ $\oplus$ $M = \{[x;y;z] \in E_3;\ 0 \leq z \leq 4 - x^2 - y^2\}$

① $\operatorname{div}\vec{f} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = \cancel{(1+1+0)}\ 1 + 1 + 0 = 2$ $\quad J = r$

② G-O. věta: $\iint_\sigma \vec{f} \cdot d\vec{p} = \iiint_M \operatorname{div}\vec{f}\ dx\,dy\,dz = \iiint 2\ dx\,dy\,dz =$

$$= \int_0^2\int_0^{2\pi}\int_0^{4-x^2-y^2} 2r\ dz\,du\,dr = \int_0^2 2r\int_0^{2\pi} [z]_0^{4-x^2-y^2}\, du\,dr = \int_0^2 2r\int_0^{2\pi} (4 - x^2 - y^2)\,du\,dr =$$

$$= \int_0^2 2r\int_0^{2\pi} 4 - r^2\cos^2 u - r^2\sin^2 u\ du\,dr = \int_0^2 2r\int_0^{2\pi} (4 - r^2)\,du\,dr =$$

$$= \int_0^2 (8r - 2r^3) \cdot [u]_0^{2\pi} = 2\pi \cdot \left[\frac{8r^2}{2} - \frac{2r^4}{4}\right]_0^2 = 2\pi \cdot (16 - 8) = \underline{\underline{16\pi}}$$

MATEMATIKA: 14/15

① $f(x) = e^{3x-6} - x$

a) tečna v bodě $[x_0; f(x_0)]$; $x_0 = 2$

$t: y = f'(x_0) \cdot (x - x_0) + y_0$

$f'(x) = e^{3x-6} \cdot 3 - 1 \rightarrow f'(x_0) = 3e^{6-6} - 1 = \cancel{3e-1}\ 3 - 1 = 2$

$f(x_0) = e^{3 \cdot 2 - 6} - 2 = 1 - 2 = -1 \rightarrow \cancel{[2; e-2]}\ [2; -1]$

$\cancel{t: y = 3e-1}$ $t: y = 2 \cdot (x-2) + (-1)$

$y = 2x - 4 - 1$

$y = 2x - 5$

b) $T_2(x)$; $x_0 = 2$; hodnota v $x = \frac{5}{3}$

$T_2(x) = f(x_0) + \frac{f'(x_0)}{1!} \cdot (x - x_0) + \frac{f''(x_0)}{2!} \cdot (x - x_0)^2$

$f''(x) = 3e^{3x-6} \cdot 3 - 0 = 9e^{3x-6}$

$f''(x_0) = 9e^0 = 9$

$T_2(x) = -1 + \frac{2}{1} \cdot (x-2) + \frac{9}{2} \cdot (x-2)^2 = \cancel{-1} + 2x - 5 + \frac{9}{2}(x-2)^2$

$T_2\left(\frac{5}{3}\right) = \cancel{-1+2\cdot\frac{5}{3}-4} \frac{10}{3} - 5 + \frac{9}{2} \cdot \left(-\frac{1}{3}\right)^2 = \frac{10-15}{3} + \frac{9}{2} \cdot \frac{1}{9}$

$T_2\left(\frac{5}{3}\right) = -\frac{5}{3} + \frac{1}{2} = \cancel{-1} - \frac{10}{6} + \frac{3}{6} = -\frac{7}{6} \rightarrow f\left(\frac{5}{3}\right) \doteq -\frac{7}{6}$

c) $R_3(x)$; $R_3\left(\frac{5}{3}\right)$

$R_3(x) = \frac{27e^{3\xi-6}}{6} \cdot (x-2)^3$

$R_3\left(\frac{5}{3}\right) = \frac{\cancel{27} \cdot e^{3 \cdot \xi - 6}}{6} \cdot \frac{1}{\cancel{27}} \rightarrow R_3\left(\frac{5}{3}\right) \leq \left| \frac{e^{3 \cdot 2 - 6}}{6} \right| = \left| \frac{1}{6} \right|$

③ hom. d. ur. $\ddot{x} + 2\dot{x} - 3x = 0$

a) FS, OR, zad. Cauchy $x(0)=6$, $\dot{x}(0)=0$

$\lambda^2 + 2\lambda - 3 = 0 \rightarrow \lambda_1 = 1;\ \lambda_2 = -3$

FS: $\{\varphi_1(t) = e^{t};\ \varphi_2(t) = e^{-3t}\}$ ✓

OR: $x(t) = c_1 e^{t} + c_2 e^{-3t}$; $t \in (-\infty; \infty)$ ✓

Cauchy: $\dot{x}(t) = c_1 e^{t} - 3c_2 e^{-3t}$

$6 = 3c_2 + c_2 = 4c_2$

$6 = c_1 e^0 + c_2 e^0 \rightarrow 6 = c_1 + c_2 \rightarrow c_2 = \frac{6}{4} = \frac{3}{2}$

$0 = c_1 e^0 - 3c_2 e^0 \rightarrow 0 = c_1 - 3c_2 \rightarrow c_1 = 3c_2$

$c_1 = 3 \cdot \frac{3}{2}$

$c_1 = \frac{9}{2}$

$x(t) = \frac{9}{2} e^{t} + \frac{3}{2} e^{-3t}$; $t \in (-\infty; \infty)$

b) $\ddot{x} + 2\dot{x} - 3x = 15e^{-3t} \rightarrow f(t) = 15e^{-3t}$

$\alpha = -3;\ \beta = 0 \rightarrow \omega = \alpha + \beta i = -3 = \lambda_2 \neq \lambda_1 \rightarrow s = 1$

$x_p = e^{-3t} \cdot (A\cos 0t + B\sin 0t) \cdot t^1$

$x_p = A t e^{-3t}$

$\dot{x}_p = A \cdot (1 \cdot e^{-3t} + t \cdot (-3) \cdot e^{-3t}) = Ae^{-3t} - 3At e^{-3t}$

$\ddot{x}_p = -3Ae^{-3t} - 3A(e^{-3t} - 3te^{-3t})$

$= -3Ae^{-3t} - 3Ae^{-3t} + 9Ate^{-3t} = 9Ate^{-3t} - 6Ae^{-3t}$

$\Rightarrow \cancel{9Ate^{-3t}} - 6Ae^{-3t} + 2Ae^{-3t} - \cancel{6Ate^{-3t}} - \cancel{3Ate^{-3t}} = 15e^{-3t}$

$-4A\cancel{e^{-3t}} = 15\cancel{e^{-3t}}$

$A = -\frac{15}{4}$

$t \in (-\infty; \infty)$

$x_p(t) = -\frac{15}{4} t e^{-3t}$ $\Rightarrow$ ~~OR NR: $x(t) = \frac{9}{2}e^{t} + \frac{3}{2}e^{-3t}$~~

OR NR: $c_1 e^{t} + c_2 e^{-3t} - \frac{15}{4} t e^{-3t}$

②

MATEMATIKA : 15/16

① a) $f(x) = 2\cdot\sqrt{x-1} - x$; tečnu v $[x_0; f(x_0)]$; $x_0 = 5$

$\rightarrow$ hodnota $f(x)$ v $x = \frac{11}{2}$

$t: \; y = f'(x_0)\cdot(x - x_0) + y_0$

$f(x) = 2\cdot(x-1)^{\frac{1}{2}} - x$

$f(x_0) = 2\cdot\sqrt{4} - 5$

$f(x_0) = 4 - 5 = -1 = y_0$

$[5 ; -1]$

$f'(x) = \cancel{2}\cdot\frac{1}{\cancel{2}}\cdot(x-1)^{-\frac{1}{2}}\cdot 1 - 1 = \frac{1}{\sqrt{x-1}} - 1$

$f'(x_0) = \frac{1}{\sqrt{5-1}} - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$

$f''(x) = -\frac{1}{2}(x-1)^{-\frac{3}{2}} = -\frac{1}{2\sqrt{(x-1)^3}}$

$f''(x_0) = -\frac{1}{2\cdot\sqrt{4^3}} = -\frac{1}{16}$

$\cancel{T_2(x)}$ $t: \; y = -\frac{1}{2}\cdot(x-5) \cancel{+} - 1$

$y = -\frac{1}{2}x + \frac{5}{2} - \frac{2}{2}$

$\underline{\underline{y = -\frac{1}{2}x + \frac{3}{2}}} \rightarrow f(11/2) = -\frac{1}{2}\cdot\left(\frac{11}{2}\right) + \frac{3}{2} = -\frac{11}{4} + \frac{6}{4} = \underline{\underline{-\frac{5}{4}}}$

$T_2(x) = -1 + \frac{-\frac{1}{2}}{1}\cancel{\cdots}\cdot(x-5) + \frac{-\frac{1}{16}}{2}\cdot(x-5)^2$

$T_2(x) = -1 - \frac{x}{2} + \frac{5}{2} \cancel{\cdots} - \frac{1}{32}\cdot(x-5)^2$

$T_2(11/2) = -1 - \frac{11}{4} + \frac{5}{2} - \frac{1}{32}\cdot(11/2 - 5)^2$

$T_2(11/2) = -\frac{32}{32} - \frac{88}{32} + \frac{80}{32} - \frac{1}{32}\cdot\frac{1}{4}$

AHA, TO NEMÁM DĚLAT

b)

b) existence absolut. extrémů na intervalu $I = \langle 1; 6 \rangle$

-> polohu, typ a hodnotu

=> 1) KRITICKÉ BODY -> krajní body

-> $f'(x) = 0$

-> bod kde f' neexistuje

• interval je uzavřený a omezený ✓

-> $f(1) = 2 \cdot \sqrt{1-1} - 1 = \underline{\underline{-1}}$

$f(6) = 2 \cdot \sqrt{6-1} - 6 = \underline{\underline{2\sqrt{5} - 6}}$

-> $f'(x) = 0$ -> $\cancel{f'(x)} = f'(x) = \frac{1}{\sqrt{x-1}} - 1$

$$\frac{1}{\sqrt{x-1}} - 1 = 0$$

$$\frac{1}{\sqrt{x-1}} = 1$$

$$\sqrt{x-1} = 1$$

$$x - 1 = 1$$

$$x = 2$$

-> $f(2) = 2 \cdot \sqrt{2-1} - 2 = \underline{\underline{0}}$

$$\boxed{\begin{aligned} \max f &= f(2) = 0 \\ \min f &= f(6) = 2\sqrt{5} - 6 \end{aligned}}$$ ✓ ✓

MAT. 15/16

(2) a) $A = \begin{pmatrix} 3 & 5 & 2 \\ -1 & 5 & 3 \\ 0 & 0 & 2 \end{pmatrix}$

$$\det(A-\lambda E) = \begin{vmatrix} 3-\lambda & 5 & 2 \\ -1 & 5-\lambda & 3 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (3-\lambda)\cdot(5-\lambda)\cdot(2-\lambda)+0+0-0-0-(5\lambda-10) =$$

$$(-5\cdot(2-\lambda))$$

$$= (15-8\lambda+\lambda^2)\cdot(2-\lambda) - \cancel{5\lambda+10} = \cancel{30-15\lambda-16\lambda+8\lambda^2-\lambda^3-2\lambda^2+5\lambda+10} =$$

$$\cancel{= -\lambda^3+10\lambda^2-36\lambda+40}$$

$$\cancel{-\lambda^3+10\lambda^2-36\lambda=-40}$$

$$\cancel{(15-8\lambda+\lambda^2)\cdot(2-\lambda)-5\cdot(2-\lambda)=0}$$

$$\cancel{\lambda^2-8\lambda+10=0}$$

$$= (2-\lambda)\cdot(\lambda^2-8\lambda+20)=0$$

$\Downarrow$ $\underline{\underline{\lambda_1 = 2}}$

$\Downarrow$ $D = 64-4\cdot1\cdot20 = -16 = 16i^2$

$$\lambda_{2,3} = \frac{8\pm\sqrt{16i^2}}{2} = \begin{cases} \dfrac{8+4i}{2} = \underline{\underline{4+2i}} \\ \dfrac{8-4i}{2} = \underline{\underline{4-2i}} \end{cases}$$

b) $\boxed{\lambda_1 = 2}$

$$\begin{pmatrix} 1 & 5 & 2 \\ -1 & 3 & 3 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 8 & 5 \\ -1 & 3 & 3 \\ 0 & 0 & 0 \end{pmatrix} \to 8y = -5z$$

$\Downarrow$

$y = -5;\ z = 8$

$\Rightarrow -x+3y+3z = 0$

$-x-15+24=0 \to x = 9$

$$\boxed{\vec{u} = \begin{pmatrix} 9 \\ -5 \\ 8 \end{pmatrix} t\ ;\ t\in\mathbb{R}-\{0\}}$$

(3) nebo $\vec{u} = (9t; -5t; 8t)^T\ t\in\mathbb{R}, t\neq 0$

③ autonomní soust. nelin. dif. rovnic

$\dot{x} = 2y(x+2)$; $\dot{y} = x^2 - y^2$ = $Q(x,y)$

$\dot{x} = 2xy + 4y = P(x,y)$

$\dot{y} = x^2 - y^2 = Q(x,y)$

$P(x,y)$ // $\cancel{\dot{x} = f_1(y) = 2y(x+2) = P(y)}$

$\cancel{\dot{y} = f_1(x) = 2x^2 - y^2 = Q(x)}$

a) $\boxed{J = \begin{pmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial y} \\ \frac{\partial Q}{\partial x} & \frac{\partial Q}{\partial y} \end{pmatrix}} = \begin{pmatrix} 2y & 2x+4 \\ 2x & -2y \end{pmatrix}$ -> je spojitá v E_2.

b) body rovnováhy -> položím $P(x,y)=0$ a $Q(x,y)=0$

$\cancel{2xy + 4y = 0}$

• $2y(x+2) = 0$

$y = 0$; $x = -2$

• $x^2 - y^2 = 0$

$x^2 = y^2$

pro $x = -2$ => $y^2 = 4$

$y = \pm 2$

pro $y = 0$ -> $x = 0$

=> B.R.: $[0;0]$; $[-2;-2]$; $[-2;2]$

c) rovnice fázových trajektorií ; následně r. f. t. procházející $M = [3;1$

$\boxed{\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}}$ $\frac{dy}{dx} = \frac{x^2 - y^2}{2y(x+2)}$ => $\int 2y(x+2)\,dy = \int (x^2 - y^2)\,dx$ ->

-> $2(x+2) \cdot \int y = \int (x^2 - y^2)\,dx$

$\cancel{2}(x+2) \cdot \frac{y^2}{\cancel{2}} = \frac{x^3}{3} - xy^2 + C$

$xy^2 + 2y^2 = \frac{x^3}{3} - xy^2 + C$

$2xy^2 + 2y^2 - \frac{x^3}{3} - C = 0$

$\frac{dy}{dx} = \frac{y'}{x} \rightarrow \frac{dy}{dx} = \frac{x^2 - y^2}{2y(x+2)} \rightarrow 2y(x+2)\,dy = (x^2 - y^2)\,dx$

$2y(x+2)\,dy + (y^2 - x^2)\,dx = 0$

- $\frac{dh}{dx} = y^2 - x^2$ • $\frac{dh}{dy} = 2y(x+2)$

1) $\int (2yx + 4y)\,dy = y^2 x + 2y^2 + h(x)$

2) der. dle x $\rightarrow y^2 + \frac{h(x)}{dx} = \frac{dh}{dx}$

$y^2 + \frac{h(x)}{dx} = y^2 - x^2$

$\frac{h(x)}{dx} = -x^2 \rightarrow h(x) = -x^2 dx$

$h(x) = \int -x^2\,dx = -\frac{x^3}{3} + c$

dosadit do 1) $\rightarrow$ $y^2 x + 2y^2 - \frac{x^3}{3} + C = 0$

bod $M = [3;1]$

$1^2 \cdot 3 + 2 \cdot 1^2 - \frac{3^3}{3} + C = 0$

$3 + 2 - 9 + C = 0$

$C = 9 - 5$

$C = 4$

$\Rightarrow$ $y^2 x + 2y^2 - \frac{x^3}{3} + 4 = 0$

MATEMATIKA: 16/17

① $f(x) = (x-6)\sqrt{x} = (x-6)\cdot x^{\frac{1}{2}} = x^{\frac{3}{2}} - 6x^{\frac{1}{2}}$

a) $D(f)$; tečnu v $[x_0, f(x_0)]$; $x_0 = 4$; $f(x)$ pro $x = \frac{9}{2}$

$D(f) = \langle 0; \infty)$

$f(x_0) = (4-6)\sqrt{4} = -2\cdot 2 = -4 = y_0$

$t: y = f'(x)\cdot(x - x_0) + y_0$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}} - \frac{1}{2}\cdot 6x^{-\frac{1}{2}} = \frac{3\sqrt{x}}{2} - \frac{3}{\sqrt{x}}$

$t: y = \left(\frac{3\sqrt{x_0}}{2} - \frac{3}{\sqrt{x_0}}\right)\cdot(x-4) \cancel{+} -4$

$y = \left(\frac{3\cdot 2}{2} - \frac{3}{2}\right)\cdot(x-4) - 4$

$y = \cancel{3x}\ \frac{3}{2}x - \frac{4\cdot 3}{2} - 4$

$y = \frac{3}{2}x - 10$ $\rightarrow$ $f(\cancel{x}\ \frac{9}{2}) = \frac{3}{2}\cdot\frac{9}{2} - 10$

$f\left(\frac{9}{2}\right) = \frac{27}{4} - 10 = -\frac{13}{4}$

b) $I = \langle 0; 4\rangle$

$\hookrightarrow$ fce je spojitá na daném intervalu

I je uzavřený a omezený

- $f(0) = (0-6)\cdot\sqrt{0} = 0$
- $f(4) = (4-6)\cdot\sqrt{4} = -2\cdot 2 = -4$
- $\frac{3\sqrt{x}}{2} - \frac{3}{\sqrt{x}} = 0$

$\frac{3\sqrt{x}}{2} = \frac{3}{\sqrt{x}}$ $\rightarrow$ $\frac{3x}{2} = 3$

$\frac{x}{2} = 1$

$x = 2$

$f(2) = (2-6)\cdot\sqrt{2} = -4\sqrt{2}$

$\boxed{\max f = f(0) = 0;\ \min f = f(2) = -4\sqrt{2}}$

①

2) $\ddot{x} + 3\dot{x} + 2x = 5e^{-2t}$

a) $\ddot{x} + 3\dot{x} + 2x = 0$

$\lambda^2 + 3\lambda + 2 = 0$

$\lambda_2 = -1 \quad \lambda_1 = -2$

FS: $\{\varphi_1(t) = e^{-2t} ; \varphi_2(t) = e^{-t}\}$

OŘ: $x(t) = C_1 e^{-2t} + C_2 e^{-t} \; ; \; t \in (-\infty; \infty)$

b) $f(t) = 5e^{-2t} \rightarrow$ polynom 0. stupně

$\alpha = -2 ; \beta = 0 \rightarrow \omega = -2 = \lambda_1 \neq \lambda_2 \rightarrow k = 1$

$x_p = e^{-2t} \cdot (A \cos 0t + \cancel{B \sin 0t}) \cdot t^1$

$x_p = A t e^{-2t}$

$\dot{x}_p = A \cdot (e^{-2t} - 2t e^{-2t}) = A e^{-2t} - 2At e^{-2t}$

$\ddot{x}_p = -2Ae^{-2t} - 2A \cdot (e^{-2t} - 2t e^{-2t}) = -4Ae^{-2t} + 4Ate^{-2t}$

$-4Ae^{-2t} + \cancel{4Ate^{-2t}} + 3Ae^{-2t} - \cancel{6Ate^{-2t}} + \cancel{2Ate^{-2t}} = 5e^{-2t}$

$-A\cancel{e^{-2t}} = 5\cancel{e^{-2t}}$

$A = -5 \rightarrow x_p = -5te^{-2t}$

OŘNR: $x(t) = C_1 e^{-2t} + C_2 e^{-t} - 5te^{-2t} \; ; \; t \in (-\infty; \infty)$

c) $x(0) = 5 ; \dot{x}(0) = -15 \rightarrow \dot{x}(t) = -2C_1 e^{-2t} - C_2 e^{-t}$

$5 = C_1 \cdot e^0 + C_2 e^0 \rightarrow 5 = C_1 + C_2 \rightarrow C_1 = 5 - C_2$

$-15 = -2 \cdot C_1 e^0 - C_2 e^0 \rightarrow -15 = -2C_1 - C_2$

$-15 = -2 \cdot (5 - C_2) - C_2$

$-15 = -10 + 2C_2 - C_2$

$-5 = C_2 \rightarrow C_1 = 5 - (-5)$

$C_1 = 10$

$x(t) = 10e^{-2t} - 5e^{-t} - 5te^{-2t}$

$\boxed{x(t) = 5e^{-2t} - 5e^{-t}}$

3)

$$\dot{x} = x^2 + y^3 = P(x;y)$$

$$\dot{y} = -2x(y+1) = Q(x;y) \quad \rightarrow -2xy - 2x$$

a)

$$J = \begin{pmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial y} \\ \frac{\partial Q}{x} & \frac{\partial Q}{y} \end{pmatrix} = \begin{pmatrix} 2x & 3y^2 \\ -2y-2 & -2x \end{pmatrix}$$

je spojitá v E_2 ✓

b)

- $x^2 + y^3 = 0$
- $-2x(y+1) = 0$

(I) $x = 0$ (II) $y = -1$

(I) $y^3 = 0$

$y = 0 \Rightarrow [0;0]$ ✓

(II) $x^2 + (-1)^3 = 0$

$x^2 = 1$

$x = \pm 1 \rightarrow [-1;-1]$

$[1;-1]$

c) fáz. kr.; $M = [-2;3]$

$$\frac{dy}{dx} = \frac{Q(x,y)}{P(x,y)} \rightarrow \frac{dy}{dx} = \frac{-2xy-2x}{x^2+y^3} \rightarrow (x^2+y^3)\,dy = (-2xy-2x)\,dx$$

$$\rightarrow (x^2+y^3)\,dy + (2xy+2x)\,dx = 0$$

- $\frac{dh}{dy} = x^2 + y^3$
- $\frac{dh}{dx} = 2xy + 2x$

1) $\int (2xy + 2x)\,dx = x^2y + x^2 + h(y)$

2) $\rightarrow$ der. dle $y \rightarrow x^2 + \frac{h(y)}{dy} = \frac{dh}{dy}$

$$x^2 + y^3 = x^2 + \frac{h(y)}{dy}$$

$\downarrow$

$$y^3 = \frac{k_y}{d_y}$$

$$k_y = \int y^3 \, dy = \frac{y^4}{4} + C$$

DOSADIT: $x^2 y + x^2 + \frac{y^4}{4} + C = 0$

$\rightarrow M = [-2; 3]$

$$(-2)^2 \cdot 3 + (-2)^2 + \frac{3^4}{4} + C = 0$$

$$12 + 4 + \frac{81}{4} + C = 0$$

$$\frac{48 + 16 + 81}{4} + C = 0$$

$$C = -\frac{145}{4}$$

$$x^2 y + x^2 + \frac{y^4}{4} = \frac{145}{4}$$

ČMS

① a) $d = ?$

$M_k = 92\ Nm$

$\tau = 30\ N/mm^2$

$$d = \sqrt[3]{\frac{16 M_k}{\pi \cdot \tau_D}} = \sqrt[3]{\frac{16 \cdot 92000}{\pi \cdot 30}} = \underline{\underline{25\ mm}}$$

b)

TE 1 : $D = 250\ mm$; $L = 1\ mm$; $R_m = 310\ MPa$; $R_{p0,2} = 140\ MPa$

$\rightarrow$ 2 tahy ; $m_1 = 0{,}6$; $m_2 = 0{,}8$

$$\boxed{d_n = D \cdot m_n}$$

1) kosinec, kosnice, přidržovač

2) $d_1 = D \cdot m_1 = 250 \cdot 0{,}6 = \underline{\underline{150\ mm}}$

$d_2 = d_1 \cdot m_2 = 150 \cdot 0{,}8 = \underline{120\ mm}$

3) $V_0 = V_2$

$$\frac{\pi D^2}{4} \cdot 1 = \left(\frac{\pi \cdot 120^2}{4} - \frac{\pi \cdot 118^2}{4}\right) \cdot h$$

$$\boxed{h = \frac{D^2 - d_n^2}{4 \cdot d_n}} = \frac{250^2 - 120^2}{4 \cdot 120} = \underline{\underline{100{,}2\ mm}}$$

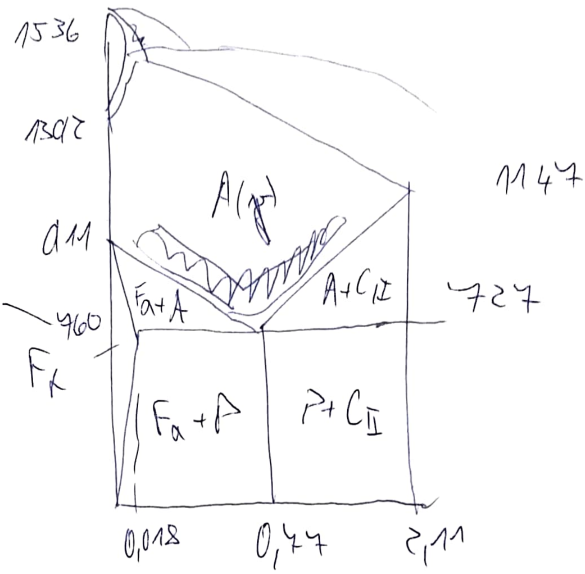
$$h = \frac{\frac{\pi \cdot 250^2}{4} \cdot 1}{\frac{\pi \cdot 120^2}{4} - \frac{\pi \cdot 118^2}{4}} = \frac{250^2}{120^2 - 118^2} = 131\ ?\ \text{chybne}$$

4) $$\boxed{F = R_m \cdot \pi \cdot d_1 \cdot 1} = 310 \cdot \pi \cdot 150 \cdot 1 = \underline{\underline{146\ kN}}$$

NoM

① C55 -> Ušlechtilá (neleg. + C)

* nekalitelné oceli pod 0,03% C, dále oceli feritické a austenit…

② 

• normalizování
↳ 800-1200°C - homogenizace
- jemnozrnnost
-> po kování, válcování za tepla či slévání

③ $d_0 = 14\,mm$; $F_e = 53{,}2\,kN$, $F_m = 101{,}1\,kN$

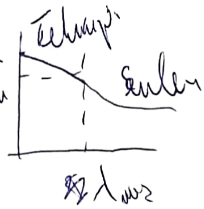
a) $R_e = \dfrac{F_e}{S_0} = \dfrac{53200}{\frac{\pi \cdot 14^2}{4}} = \underline{\underline{346\,MPa}}$

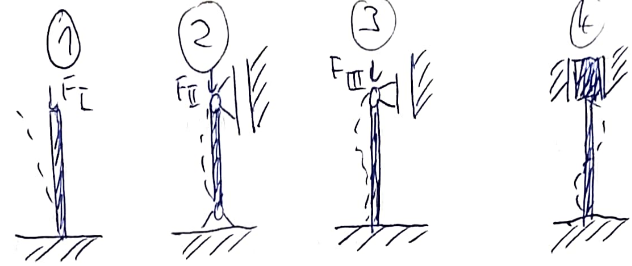
b) $R_m = \dfrac{F_m}{S_0} = \dfrac{101100}{\frac{\pi \cdot 14^2}{4}} = \underline{\underline{657\,MPa}}$

A_{min} -> na šikmé tyči

PP

VZPĚR



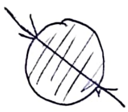


$n = \frac{1}{4}$ $n = 1$ $n = 2$ $n = 4$

• průřez


a: $A = a^2$; $J_{min} = \frac{a^4}{12}$


b, h: $A = b \cdot h$; $J_{min} = \frac{b \cdot h^3}{12}$


$A = \frac{\pi \cdot d^2}{4}$; $J_{min} = \frac{\pi \cdot d^4}{64}$

$\sigma > \sigma_m \Rightarrow$ Tetmajer

$\sigma < \sigma_m \Rightarrow$ Euler

• $i_{min} = \sqrt{\frac{J_{min}}{A}}$

• $\lambda = \frac{l}{i_{min}}$ ... štíhlost prutu

• $\lambda_{mez} = \sqrt{\frac{n \cdot \pi \cdot E}{\sigma_m}}$ ... mezní štíhlost

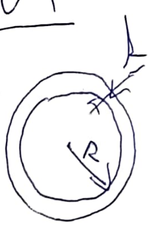
• $F_{KR}^E = k \cdot F_I = \frac{n \cdot \pi^2 \cdot E \cdot J_{min}}{l^2}$

• $\sigma_{KR}^E = \frac{F_{KR}}{A} = \frac{k \cdot F}{A}$ $\rightarrow \sigma_{KR}^E < \sigma_m$; $\lambda^E > \lambda_{mez}$

• $\sigma_{KR}^T = \sigma_d - (\sigma_d - \sigma_m) \cdot \frac{\lambda}{\lambda_{mez}}$ $\rightarrow \sigma_{KR}^T \in \langle \sigma_m ; \sigma_K \rangle$

$\lambda^T < \lambda_{mez}$

KRUT

• 

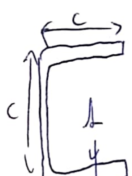
$h = 2 \cdot \pi \cdot R$

$J_K = \frac{2}{3} \pi \cdot R \cdot t^3$

• 

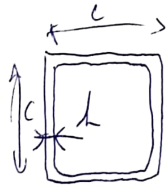
$h = \pi \cdot R$

$J_K = \frac{1}{3} \pi \cdot R \cdot t^3$

• 

$h = 3 \cdot c$

$J_K = \frac{1}{3} \cdot 3 \cdot c \cdot t^3$

• 

$W_k = 2 \cdot A_s \cdot t_{min}$

$W_k = 2 \cdot c^2 \cdot t$

$J_K = \frac{1}{3} \cdot \text{obvod} \cdot t^3$

$\uparrow h$

PRŮŘEZOVÉ CHARAKTERISTIKY:

$J_K = \frac{1}{3} h t^3$ $W_K = \frac{1}{3} h t^2$

PEVNOSTNÍ PODMÍNKA: $\tau_{MAX} = \frac{M_K}{W_K} \leq \tau_D$

NATOČENÍ KONCŮ: $\varphi_{A-B} = \frac{M_K \cdot l}{G \cdot J_K} = [RAD] \rightarrow \cdot \frac{180}{\pi} [°]$

(Pr4) kruhový průřez d, l

→ 3. případ uložení → $n = 2$

D: $\sigma_K = R_e = 260\ N/mm^2$
$\sigma_M = R_M = 200\ N/mm^2$
$E = 2 \cdot 10^5\ N/mm^2$
$l = 600\ mm$
$d = 12\ mm$
$k_E = 3$

U: ① F_{KR}, σ_{KR}
② F_D ; σ_D

① $$F_{KR} = \frac{n \cdot \pi^2 \cdot E \cdot J_{min}}{l^2} = \frac{n \cdot \pi^2 \cdot E \cdot \frac{\pi d^4}{64}}{l^2} = \frac{2 \cdot \pi^2 \cdot 2 \cdot 10^5 \cdot \frac{\pi \cdot 12^4}{64}}{600^2} = 11162{,}26\ N$$

$$\sigma_{KR} = \frac{F_{KR}}{A} = \frac{11162{,}26}{\frac{\pi \cdot 12^2}{4}} = 98{,}7\ N/mm^2 < \sigma_M \Rightarrow \text{platí Euler}$$

② $$F_D = \frac{F_{KR}}{k_E} = \frac{11162{,}26}{3} = 3720{,}75\ N$$

$$\sigma_D = \frac{F_D}{A} = \frac{3720{,}75}{\frac{\pi \cdot 12^2}{4}} = 32{,}9\ N/mm^2$$

(Pr2) D: $h = 130\ mm$, $l = 1200\ mm$, $t = 4\ mm$, $R = 15\ mm$, $G = 0{,}8 \cdot 10^5\ N/mm^2$,
$M_K = 2 \cdot 10^4\ N \cdot mm$ ($a = 25\ mm$, $b = 33\ mm$)

U: ① τ_{max} ; ② φ_{A-B}

$$J_K = \frac{1}{3} h \cdot t^3 = \frac{1}{3} \cdot 130 \cdot 4^3 = 2773{,}3\ mm^4$$

$$W_K = \frac{1}{3} h \cdot t^2 = \frac{1}{3} \cdot 130 \cdot 4^2 = 693{,}3\ mm^3$$

$$\tau_{max} = \frac{M_K}{W_K} = \frac{2 \cdot 10^4}{693{,}3} = 28{,}85\ N/mm^2$$

$$\varphi_{A-B} = \frac{M_K \cdot l}{G \cdot J_K} = \frac{2 \cdot 10^4 \cdot 1200}{0{,}8 \cdot 10^5 \cdot 2773{,}3} = 0{,}108\ rad \approx 6{,}2^\circ$$

MATEMATIKA : 17/18

1) $f(x) = \frac{x^2+4}{x} + 2 = x + \frac{4}{x} + 2$ $\to y_0 = 4 + \frac{4}{4} + 2$

$y_0 = 4+3 = 7$

a) D_f, tečna, $x_0=4$, $f(\frac{7}{2})$

$D(f) = (-\infty; 0) \cup (0; \infty)$ ✓

$t: y = f'(x_0) \cdot (x - x_0) + y_0$

$f'(x) = 1 + (-1) \cdot 4 \cdot x^{-2} + 0 = 1 - \frac{4}{x^2}$

$t: y = \left(1 - \frac{4}{4^2}\right) \cdot (x-4) + 7$

$y = \frac{3}{4}x - 3 + 7$

$y = \frac{3}{4}x + 4$ ✓ $\to f(\frac{7}{2}) = \frac{3}{4} \cdot \frac{7}{2} + 4 = \frac{21}{8} + \frac{32}{8}$

$= \frac{53}{8}$

b) $I = \langle -3; -1 \rangle$

$\to$ spojitá; I je uzavř. a omez.

- $f(-3) = \frac{9+4}{-3} + 2 = \frac{13-6}{-3} = -\frac{7}{3}$
- $f(-1) = \frac{1+4}{-1} + 2 = -5 + 2 = -3$
- $1 - \frac{4}{x^2} = 0$

$\frac{4}{x^2} = 1$

$x^2 = 4$

$x = \pm 2$

$\to f(-2) = \frac{4+4}{-2} + 2 = -4 + 2 = -2$

~~$f(2) = \frac{4+4}{2} + 2 = 6$~~

+2 není v intervalu

$\Rightarrow$ $\min f = f(-1) = -3$; $\max f = f(-2) = -2$

② $A = \begin{pmatrix} 1 & 2 & -5 \\ 0 & 2 & 0 \\ 1 & 2 & 3 \end{pmatrix}$

$\boxed{\det(A - \lambda E) = 0}$

a) $\det(A-\lambda E) = \begin{vmatrix} 1-\lambda & 2 & -5 \\ 0 & 2-\lambda & 0 \\ 1 & 2 & 3-\lambda \end{vmatrix} = (1-\lambda)\cdot(2-\lambda)\cdot(3-\lambda)+0+0+5\cdot(2-\lambda)-0-0=$

$= \cancel{(2-3\lambda+\lambda^2)}\ (3-4\lambda+\lambda^2)\cdot(2-\lambda)-5\cdot(2-\lambda)$

$(2-\lambda)\cdot(\lambda^2-4\lambda+8)=0$

$\lambda_1 = 2$

$D = 16 - 4\cdot 1\cdot(8) = -16 = 16i^2$

$\lambda_{2,3} = \dfrac{4 \pm \sqrt{16i^2}}{2} = $ $2+2i$; $2-2i$

b) $\lambda_1 = 2$

$\begin{pmatrix} -1 & 2 & -5 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{pmatrix} \sim \begin{pmatrix} 0 & 4 & -4 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{pmatrix} \to 4y = 4z$

$\Downarrow$

$y = 1$; $z = 1$

$x + 2y + z = 0$

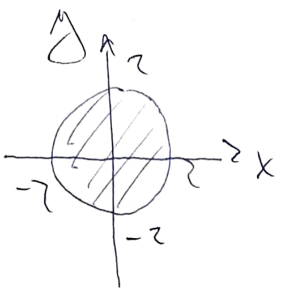
$x + 2 + 1 = 0 \to x = -3$

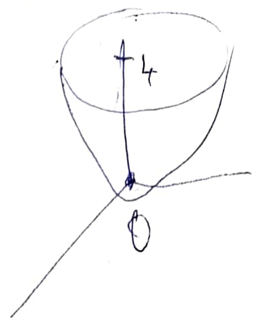
$\vec{u} = \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} k$; $k \in \mathbb{R}$, $k \neq 0 \Rightarrow \vec{u} = (-3k; k; k)^T$; $k \in \mathbb{R}$, $k \neq$

3 17/18

$Q = \{[x,y,z] \in E_3 ;\ z = x^2+y^2 ;\ z \le 4\}$

a) - výpočet plochy vůči normále $\vec{n}$
- PARAMETRIZACE, normála $\perp$ k Q
- tok $\vec{F} = (x, y, 0)$ plochou Q





$x^2 + y^2 \le 4$

$0 \le r \le 2$

$0 \le u \le 2\pi$

PARAMETRIZACE:

$x = r \cdot \cos u$

$y = r \cdot \sin u$

$z = z \quad \rightarrow z = x^2 + y^2 = r^2 \cos^2 u + r^2 \sin^2 u$

$\Downarrow$

$z = r^2$

$P(u; r) = (r \cos u ;\ r \sin u ;\ r^2)$

$P_u(u; r) = (-r \sin(u) ;\ r \cos u ;\ 0)$

$-r\sin^2(u) - r\cos^2(u)$

$P_r(u; r) = (\cos u ;\ \sin(u) ;\ 2r)$

$$\vec{n}_Q = P_u \cdot P_r = \begin{vmatrix} i & j & k \\ -r\sin(u) ; & r\cos(u) ; & 0 \\ \cos(u) & \sin(u) & 2r \end{vmatrix} = \left(2r^2\cos(u) ;\ 2r^2\sin(u) ;\ -r\right)$$

$\rightarrow$ orientace: $\vec{n}_Q \cdot (0;0;1) > 0$

$\hookrightarrow (2r^2\cos(u) ;\ 2r^2\sin(u) ;\ -r) \cdot (0;0;1) = -r$

$\Downarrow$

$-$

$\vec{f} = (x; y; 0)$

TOK: $\iint_Q \vec{f}\, d\vec{p} = -\iint_Q \vec{f} \cdot \vec{n}_Q^{\,0}\, dp =$

$$= -\int_0^2 \int_0^{2\pi} (r\cos(u); r\sin(u); 0) \cdot (2r^2\cos u; 2r^2 \sin u; -r)\, du\, dr =$$

$$= -\int_0^2 \int_0^{2\pi} (2r^3\cos^2 u + 2r^3 \sin^2 u)\, du\, dr = -\int_0^2 \int_0^{2\pi} 2r^3\, du\, dr =$$

$$= -\int_0^2 \left[2r^3 u\right]_0^{2\pi} dr = -\int_0^2 4\pi r^3\, dr = -4\pi \cdot \left[\frac{r^4}{4}\right]_0^2 =$$

$$= -\pi \cdot \left[r^4\right]_0^2 = -16\pi$$

b) divergencia div $\vec{f}$ vektorového poľa ~~(x; y; z) plochou~~

$\vec{f} = (xy^2; x+z; x^2 z)$

- pomocou G-O vety
- tok poľa $\vec{f}$ plochou σ, do vnútra $\to$ $\ominus$

$J = r$

$M = \{[x, y, z] \in E_3;\ x^2 + y^2 \le 4;\ 0 \le z \le 3\}$

• $\operatorname{div} \vec{f} = \frac{\partial \vec{f}}{\partial x} + \frac{\partial \vec{f}}{\partial y} + \frac{\partial \vec{f}}{\partial z} = y^2 + 0 + x^2 = x^2 + y^2$

• G-O veta: $\iint_\sigma \vec{f} \cdot d\vec{p} = \iiint_M \operatorname{div} \vec{f}\, dx\, dy\, dz = -\iiint (x^2 + y^2)\, dx\, dy\, dz =$

$$= -\int_0^3 \int_0^2 \int_0^{2\pi} (r^2 \cos^2 u + r^2 \sin^2 u) \cdot r \cdot du\, dr\, dz = -\iiint r^3\, du\, dr\, dz =$$

$$= -\int_0^3 \int_0^2 r^3 \cdot [u]_0^{2\pi}\, dr\, dz = -\int_0^3 \int_0^2 r^3\, 2\pi\, dr\, dz = -\int_0^3 2\pi \cdot \left[\frac{r^4}{4}\right]_0^2 dz =$$

$$= -\int_0^3 2\pi \cdot 4\, dz = -\left[8\pi z\right]_0^3 = -24\pi$$

4

TE1 – 1C ohřízel a, b, c

TE2 – prožhání CNC

$v_c = 125$ m/min

$f = 0,2$

$a_p = 2,5$

$R_m = 500$

$D = 100$

$L = 350$

a) $n = \frac{1000 \cdot v_c}{\pi \cdot D} = \frac{1000 \cdot 125}{\pi \cdot 100} = 398 \text{ min}^{-1}$

b) $A_D = f \cdot a_p = 0,2 \cdot 2,5 = 0,5 \text{ mm}^2$

c) $F_c = k_c \cdot A_D = C_k \cdot R_m \cdot A_D = 4 \cdot 500 \cdot 0,5 = 1000 N$

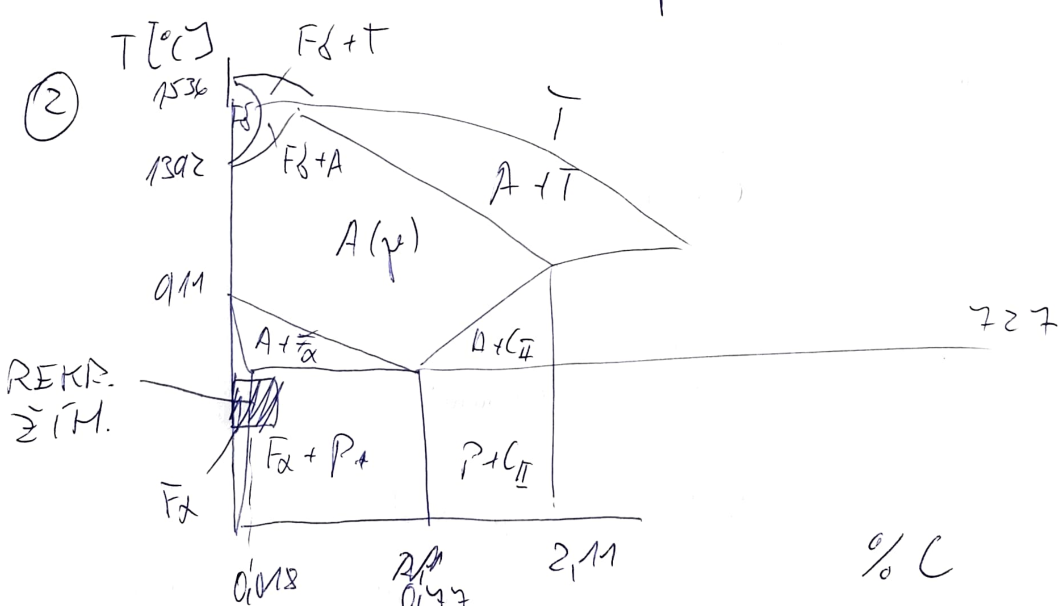
d) $P_c = F_c \cdot \frac{v_c}{60} = 1000 \cdot \frac{125}{60} = 2083 W = 2,08 kW$

e) $t_{AS} = \frac{60 \cdot L}{f \cdot n} = \frac{60 \cdot 350}{0,2 \cdot 398} = 263,8 s$

NoM – sadání A

1) EN-GJL-100 → litina s lup. grafitem → min pevnost v tahu 100 MPa

2)



oceli ← | → litiny

③ S235JR → $d_0 = 14\ mm$; $F_e = 52,7\ kN$, $F_m = 56,9\ kN$

a) $R_e = \frac{F_e}{S_0} = \frac{52700}{\frac{\pi \cdot 14^2}{4}} = \underline{\underline{342\ MPa}}$ b) $R_m = \frac{F_m}{S_0} = \frac{56900}{\frac{\pi \cdot 14^2}{4}} = \underline{\underline{433\ MPa}}$

ČMS

zadanie A → ① $m = 2\ mm$ $z_1 = 20$; $x_1 = +0,1$, $x_2 = -0,1$
$i = 3,9$

① U: $a, a_w, d_2, d_{a2}, d_{f2}$

$d_2 = m \cdot z_2 = m \cdot z_1 \cdot i = 2 \cdot 20 \cdot 3,9 = \underline{\underline{156\ mm}}$

$d_1 = m \cdot z_1 = 40\ mm$

$a = a_w = \frac{d_1 + d_2}{2} = \frac{196}{2} = \underline{\underline{98\ mm}}$

$d_{a2} = d_2 + 2m \cdot (h_a^* + x_2) = 156 + 2 \cdot 2 \cdot (1 - 0,1) = \underline{\underline{159,6\ mm}}$

$d_{f2} = d_2 - 2m \cdot (h_a^* + c^*) + 2m x_2 = 156 - 2 \cdot 2 \cdot (1 + 0,25) + 2 \cdot 2 \cdot (-0,1) = \underline{\underline{150,6}}$

② pero → $d = 25\ mm$ $b = 8$
$M_k = 90\ Nm$ $h = 7$
min $l = ?$; $p = 50\ MPa$
→ $\tau = ?$

$p = \frac{F}{S_K}$ → $S_K = \frac{F}{p} = \frac{\frac{90000}{12,5}}{50} = 144\ mm^2$

$S_K = (l - b) \cdot \frac{h}{2}$ → $\frac{2 S_K}{h} = l - b$

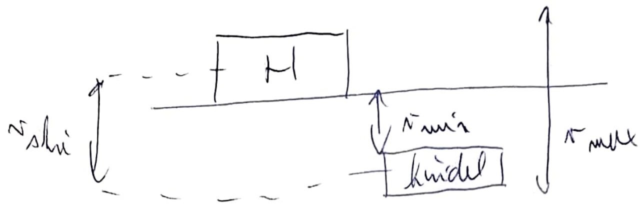
$l = \frac{2 \cdot 144}{7} + 8 = \underline{\underline{49,1\ mm}}$

$l_a = 41,1\ mm$

$S_\tau = 41,1 \cdot 8 = 328,8\ mm^2$

$\tau = \frac{\frac{90\,000}{12,5}}{328,8} = \underline{\underline{21,9\ MPa}}$

③ uložení -> volné -> j. díra



varianta B

① ozubení -> $m = 2\,mm$, $i = 4{,}1$; $z_1 = 20$; $x_1 = 0{,}1$; $x_2 = -0{,}1$

U: $a, d_w, d_2, d_{a2}, d_{f2}$,

$z_2 = 20 \cdot 4{,}1 = 82$

$d_1 = 20 \cdot 2 = 40\,mm \qquad d_2 = 82 \cdot 2 = 164\,mm$

$a = d_w = \frac{40 + 164}{2} = \underline{102\,mm}$

$d_{a2} = d_2 + 2m(h_a^* + x_2) = 164 + 2 \cdot 2 \cdot (1 - 0{,}1) = \underline{167{,}6\,mm}$

$d_{f2} = d_2 - 2m(h_a^* + c^*) + 2m x_2 = 164 - 2 \cdot 2 \cdot (1 + 0{,}25) + 2 \cdot 2 \cdot (-0{,}1) = \underline{158{,}6\,mm}$

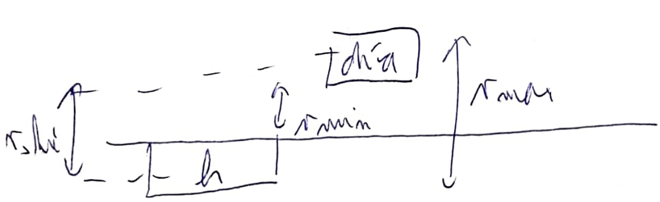
② pero -> $d = 35\,mm$, $M_k = 250\,Nm$, $p = 50\,MPa$, pero ... 10x8

$p = \frac{F}{S_k} \rightarrow S_k = \frac{\frac{250\,000}{17{,}5}}{50} = 285{,}7 \rightarrow S_k = l_a \cdot \frac{h}{2}$

$l_a = \frac{2 \cdot 285{,}7}{8} = \underline{71{,}4\,mm}$

$S_\tau = 71{,}4 \cdot 10 = 714 \rightarrow \tau = \frac{\frac{250000}{17{,}5}}{714} = \underline{20\,MPa}$

③ uložení -> volné -> j. hřídel



MATEMATIKA : 18/11

(1) $f(x)=\sqrt{2x+4} = (2x+4)^{\frac{1}{2}}$ $\rightarrow y_0 = (2\cdot 0+4)^{\frac{1}{2}}$

$y_0 = \sqrt{4} = 2$

a) $D(f)$, dotyčnica, $x_0=0$

$\cancel{\sqrt{2x+4}}$ $2x+4 \geq 0$

$2x \geq -4$ $\Rightarrow D(f) = \langle -2; \infty)$

$x \geq -2$

$$f'(x) = \cancel{\frac{1}{2}}(2x+4)^{-\frac{1}{2}} \cdot \cancel{2} = \frac{1}{\sqrt{2x+4}}$$

$$t: y = \frac{1}{\sqrt{2\cdot 0+4}}\cdot(x-0)+2$$

$$\underline{\underline{y = \frac{1}{2}x+2}}$$

b) $T_2(x)$, $x_0=0$; $f(\frac{1}{2})$

$$T_2(x) = f(x_0) + \frac{f'(x_0)}{1!}\cdot(x-x_0) + \frac{f''(x_0)}{2!}\cdot(x-x_0)^2$$

$$f''(x) = -\frac{1}{2}\cdot(2x+4)^{-\frac{3}{2}}\cdot 2 = -\frac{1}{\sqrt{(2x+4)^3}}$$

$$T_2(x) = 2 + \frac{\frac{1}{2}}{1}\cdot(x-0) + \frac{-\frac{1}{\sqrt{4\cdot 4\cdot 4}}}{2!}\cdot(x-0)^2$$

$$\boxed{T_2(x) = 2 + \frac{x}{2} - \frac{1}{16}x^2}$$

$$T_2(\tfrac{1}{2}) \doteq 2 + \frac{\frac{1}{2}}{2} - \frac{1}{16}\cdot\left(\frac{1}{2}\right)^2 = 2 + \frac{1}{4} - \frac{1}{64} = \frac{128}{64} + \frac{16}{64} - \frac{1}{64} = \underline{\underline{\frac{143}{64}}}$$

c) $R_3(x)$; $R_3(\frac{1}{2})$

$$R_3(x) = \frac{f'''(\xi)}{3!} \cdot (x - x_0)^3$$

$$f'''(x) = \frac{3}{2}(2x+4)^{-\frac{5}{2}} = \frac{3}{2\sqrt{(2x+4)^5}}$$

$$R_3(x) = \frac{\frac{3}{2\sqrt{(2\xi+4)^5}}}{6} \cdot (x-0)^3$$

$$R_3(x) = \frac{1}{2\sqrt{(2\xi+4)^5}} \cdot x^3$$

$$R_3\left(\frac{1}{2}\right) \leq \frac{1}{2\sqrt{(2\cdot 0+4)^5}} \cdot \left(\frac{1}{2}\right)^3 = \frac{1}{2^6} \cdot \frac{1}{2^3} = \frac{1}{2^9} = \underline{\underline{\frac{1}{512}}}$$

2) $A = \begin{pmatrix} 2 & -1 & 3 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$

a) spektrální poloměr $\boxed{\rho(A) = \max\{|\lambda_1|; |\lambda_2|; |\lambda_3|\}}$

$$\det(A - \lambda E) = \begin{vmatrix} 2-\lambda & -1 & 3 \\ 0 & 1-\lambda & 2 \\ 0 & 2 & 1-\lambda \end{vmatrix} = (2-\lambda)\cdot(1-\lambda)\cdot(1-\lambda) + 0 + 0 - 0 - 0 - 4\cdot(2-\lambda) =$$

$$(2-\lambda)\cdot(1-2\lambda+\lambda^2) - 4\cdot(2-\lambda) = 0$$

$$(2-\lambda)\cdot(\lambda^2 - 2\lambda - 3) = 0$$

$\lambda_1 = 2$ $\quad\lambda_2 = -1$ $\quad\lambda_3 = 3$ $\Rightarrow$ $\underline{\underline{\rho(A) = 3}}$

b) $\lambda_1 = 2$

$$\begin{pmatrix} 0 & -1 & 3 \\ 0 & -1 & 2 \\ 0 & 2 & -1 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 0 & 2 & -1 \end{pmatrix}$$

$z = 0$ $\quad x = 1$

$-y + 2z = 0$

$y = 0$

$u = (\lambda; 0; 0)^T; \lambda \in \mathbb{R}; \lambda \neq 0$

$\lambda_2 = -1$

$$\begin{pmatrix} 3 & -1 & 3 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{pmatrix} \rightarrow$$

$2y = -2z$

$y = -1$

$z = 1$

$3x - y + 3z = 0$

$3x + 1 + 3 = 0$

$3x = -4$

$x = -\frac{4}{3}$

$$\begin{pmatrix} -\frac{4}{3} \\ -1 \\ 1 \end{pmatrix} \sim \begin{pmatrix} -4 \\ -3 \\ 3 \end{pmatrix} \Rightarrow u = (-4\lambda; -3\lambda; 3\lambda)^T; \lambda \in \mathbb{R}; \lambda \neq 0$$

$\lambda_3 = 3$

$$\begin{pmatrix} -1 & -1 & 3 \\ 0 & -2 & 2 \\ 0 & 2 & -2 \end{pmatrix}$$

$2y = 2z$

$y = 1$

$z = 1$

$-x - y + 3z = 0$

$-x - 1 + 3 = 0$

$-x = -2$

$x = 2$

$u = (2\lambda; \lambda; \lambda)^T; \lambda \in \mathbb{R}; \lambda \neq 0$

3. $\dot{x} = -2y(x+1) = P(x,y) = -2yx - 2y$

$\dot{y} = x + y^2 = Q(x,y)$

$$J = \begin{pmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial y} \\ \frac{\partial Q}{\partial x} & \frac{\partial Q}{\partial y} \end{pmatrix}$$

a) $J(x,y) = \begin{pmatrix} -2y \; ; & -2x-2 \\ 1 \; ; & 2y \end{pmatrix}$ je spojitá v E_2

b) ~~[illegible]~~ $-2y(x+1) = 0$ → $y = 0$, $x = -1$

$\Rightarrow$ • $x + y^2 = 0$, $x = 0$ $\Rightarrow [0;0]$ ✓

$-1 + y^2 = 0$

$y^2 = 1$

$y = \pm 1 = [-1;1]$ ✓

$[-1;-1]$ ✓

c) fáz. krajy. ; $M = [-1;2]$

$$\frac{dy}{dx} = \frac{Q(x,y)}{P(x,y)} \rightarrow \frac{dy}{dx} = \frac{x+y^2}{-2yx-2y} \rightarrow (-2yx-2y)\,dy = (x+y^2)\,dx$$

$(x+y^2)dx + (2yx+2y)dy = 0$

$$\frac{dh}{dy} = +2yx + 2y \qquad \frac{dh}{dx} = x + y^2$$

1) $\int (2yx+2y)\,dy = y^2x + y^2 + h(x)$

→ der podle x : $y^2 + \frac{h(x)}{dx} = \frac{dh}{dx}$

$x + \cancel{y^2} = \cancel{y^2} + \frac{h_x}{dx}$

$\frac{h_x}{dx} = x \rightarrow h(x) = \int x\,dx = \frac{x^2}{2} + C$

$\Rightarrow$ rce fáz. trajektorií: $\underline{\underline{y^2x + y^2 + \frac{x^2}{2} + C = 0}}$ $\xrightarrow{M[-1;2]}$ $2^2\cdot(-1) + 2^2 + \frac{(-1)^2}{2} + C = 0$

$\cancel{-4+4} + \frac{1}{2} + C = 0$

$C = -\frac{1}{2}$

$$\boxed{y^2x + y^2 + \frac{x^2}{2} - \frac{1}{2} = 0}$$

4

TE1 – 10 jednoduchých otázek a, b, c

TE2 – pořádání CNC

$v_c = 126$ m/min

$f = 0{,}4$ mm/ot

$a_p = 2$ mm

$R_m = 700$ MPa

$D = 80$

$L = 200$

a) $n = \frac{1000 \cdot v_c}{\pi \cdot D} = \frac{1000 \cdot 126}{\pi \cdot 80} = \underline{\underline{501\ min^{-1}}}$

b) $A_D = f \cdot a_p = \underline{\underline{0{,}8\ mm^2}}$

c) $F_c = k_c \cdot A_D = c_k \cdot R_m \cdot A_D = 4 \cdot 700 \cdot 0{,}8 = \underline{\underline{2240\,N}}$

d) $P_c = F_c \cdot \frac{v_c}{60} = 2240 \cdot \frac{126}{60} = \underline{\underline{4{,}7\ kW}}$

e) $t_{AS} = \frac{60 \cdot L}{f \cdot n} = \underline{\underline{60\ s}}$

NoM

náročná práce

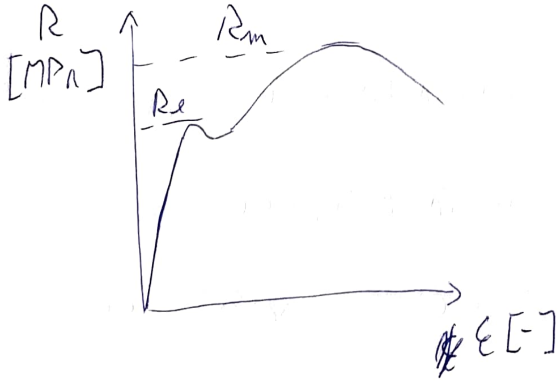
• zadání A: ① S235JR G1 -> max 0,17 % C

$d_0 = 10$ mm ; $F_e = 19{,}7$ kN , $F_m = 29{,}9$ kN , $KV_{min} = 27$ J

a) $\boxed{R_e = \frac{F_e}{S_0}} = \frac{19700}{\frac{\pi \cdot 10^2}{4}} = \underline{\underline{251\ MPa}}$ -> mez kluzu

$\boxed{R_m = \frac{F_m}{S_0}} = \frac{29900}{\frac{\pi \cdot 10^2}{4}} = \underline{\underline{380\ MPa}}$ -> mez pevnosti

b) nakreslit smluvní tahový ~~diagram~~ diagram $R = f(\varepsilon)$



c) odpovídá R_e dané oceli

-> $R_{e\,min} = 235\ MPa < R_e = 251\ MPa$ => **ANO**

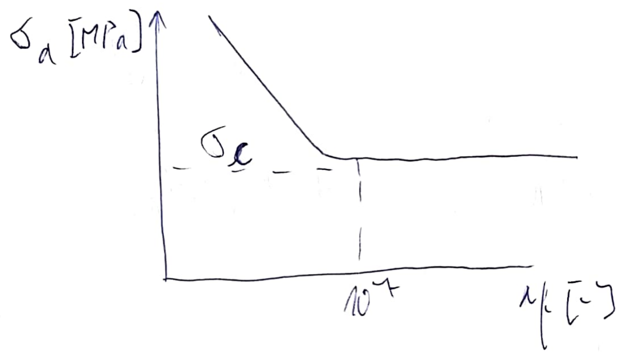
d) $L_0 = ?$ -> $L_0 = 5{,}65\sqrt{S_0} = 5d_0 = \underline{50\,mm}$?

e) ocel nemusí být kalitelná -> není legovaná a má ↓ obsah C

f) ocel je svařitelná -> nejde se do 0,22% a není legovaná

g) jak zjistit nárazovou práci? -> rázová zkouška dle Charpyho

② Wöhlerova křivka pro ocel



σ_c = amplituda napětí, které materiál snese po nekonečný počet cyklů -> stačí 10^7 cyklů

③ -> teplota rekrystalizace

mela

$[(660 + 273) \cdot 0{,}4] - 273 = \underline{100\,°C}$

na ↑ teploty ↑ poměru (0,35 – 0,45)

④ Fe_α -> stabilní do 760 °C, mřížka K8, BCC

Fe_β -> stabilní od 760 °C do 911 °C, mřížka K8, BCC, ztrácí feromag.

Fe_γ -> stabilní od 911 do 1392, mřížka K12, FCC

Fe_δ -> stabilní od 1392 do 1536 °C, mřížka K8, BCC zadání B

=> je kam 0,2 -> není výrazná mez kluzu

1. C55 -> $d_0 = 14$ mm, $l_0 = 70$ mm

$d_u = 11$ mm, $l_u = 81,2$ mm

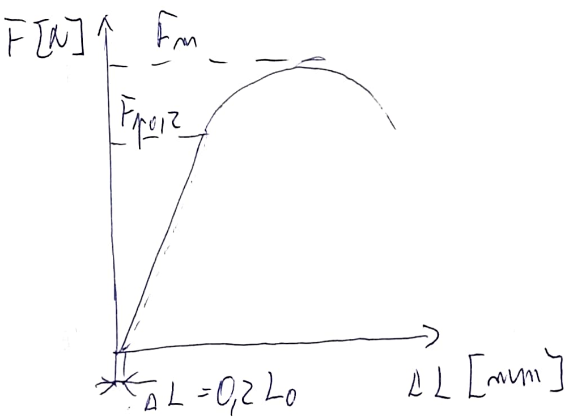
$F_{p0,2} = 63$ kN, $F_m = 120$ kN

a) $R_{p0,2} = \frac{F_{p0,2}}{S_0} = \frac{63000}{\frac{\pi \cdot 14^2}{4}} = \underline{\underline{409\ MPa}}$ $\quad R_m = \frac{F_m}{S_0} = \frac{120000}{\frac{\pi \cdot 14^2}{4}} = \underline{\underline{779\ MPa}}$

b) $\boxed{A = \frac{\Delta L}{L_0}} \cdot 100 = \frac{81,2-70}{70} \cdot 100 = \underline{\underline{16\ \%}}$ – tažnost

$\boxed{Z = \frac{\Delta S}{S_0}} \cdot 100 = \frac{\frac{\pi \cdot 14^2}{4} - \frac{\pi \cdot 11^2}{4}}{\frac{\pi \cdot 14^2}{4}} = \frac{14^2 - 11^2}{14^2} = 0,38 \rightarrow \underline{\underline{38\%}}$ kontrakce

c) tahový diagram $F = f(\Delta L)$ (pracovní)



d) -> 0,55 % C

e) zušlechťování = kalení + vysokoteplotní popouštění

f) naměřeno 248 HBW

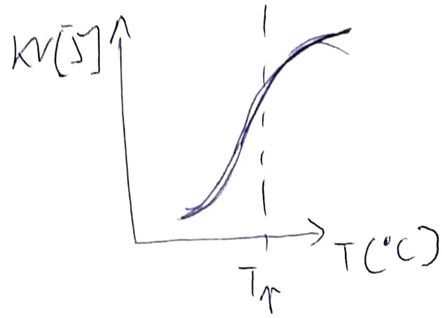
-> podle Brinella => indentor = kulička z tvrdokovu

-> zatěžovací síla závisí na st. materiálu a prům. kuličky ->

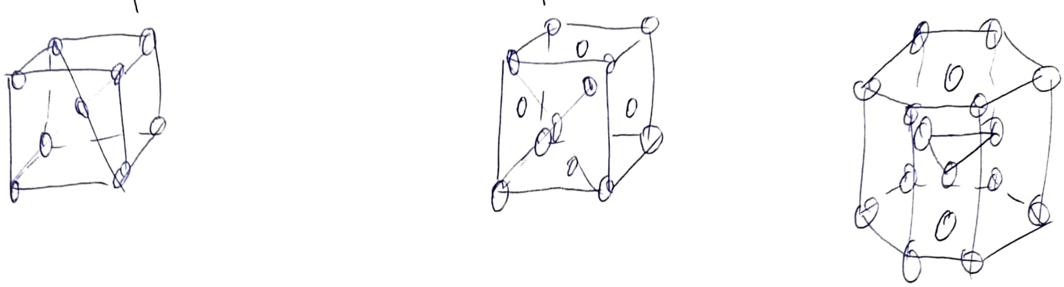
=> měří se ⌀ vtisku

7

② přechodová teplota T_p = mez. se přechází chování při rázové houževnatosti a součinu [illegible]



③ kubická prostorově (tělesně) st., k. plošně středěná, šesterečná těsně uspoř.



④ FERIT = intersticiální tuhý roztok C v Fe_α nebo Fe_β

PERLIT = směs feritu a cementitu (eutektoid metastabilní soustavy)

AUSTENIT = intersticiální t. r. C v Fe_γ

CEMENTIT = karbid železa = Fe_3C (intermediární fáze)

ČMS - varianta A

① ozubení → $m=2$, $i=3,8$, $z_1=20$, $z_2=76$, $x_1=+0,1$, $x_2=-0,1$

U: d_1, d_2, d_{a1}, d_{a2}, a_w

$d_1 = m \cdot z_1 = \underline{40\ mm}$ $\quad d_2 = m \cdot z_2 = \underline{152\ mm}$

$d_{a1} = d_1 + 2m\,(h_a^* + x_1) = 40+2\cdot 2\cdot(1+0,1) = \underline{44,4\ mm}$

$d_{a2} = d_2 + 2m\cdot(h_a^* + x_2) = 152 + 2\cdot 2\cdot(1-0,1) = \underline{155,6\ mm}$

$a_w = a = \dfrac{d_1+d_2}{2} = \dfrac{192}{2} = \underline{96\ mm}$

② pero $\rightarrow d = 28\,mm$

$M_k = 105\,Nm$

pero ... $8 \times 7 \times 30$

$$p = \frac{F}{S_K} \rightarrow F = \frac{M_k}{r} = \frac{105\,000}{14} = 7500\,N$$

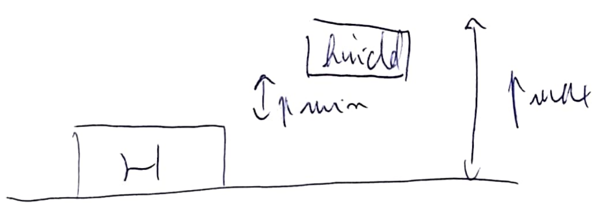
$$S_K = \frac{h}{2} \cdot (l - b) = \frac{7}{2} \cdot (30 - 8) = 77\,mm^2$$

$$\Rightarrow p = \frac{7500}{77} = 97{,}4\,MPa$$

$$\tau = \frac{F}{S_\tau} \rightarrow S_\tau = (l - b) \cdot b = 22 \cdot 8 = \cancel{154\,mm^2}\ 176\,mm^2$$

$$\tau = \frac{7500}{\cancel{154}\ 176} = \cancel{48{,}7\,MPa}\ \cancel{42{,}16\,MPa}\ 42{,}6\,MPa$$

③ uložení $\rightarrow$ přesah $\rightarrow$ jedn. díra



zadání B

① ozubení $\rightarrow m = 2;\ i = 4{,}2;\ z_1 = 20;\ z_2 = 84;\ x_1 = +0{,}1;\ x_2 = -0{,}1$

U: $d_1,\ d_2,\ d_{a1},\ d_{a2},\ a_w$

$d_1 = m \cdot z_1 = 40\,mm$ $\quad d_2 = m \cdot z_2 = 168\,mm$

$d_{a1} = d_1 + 2m(h_a^* + x_1) = 40 + 2 \cdot 2 \cdot (1 + 0{,}1) = 44{,}4\,mm$

$d_{a2} = d_2 + 2m(h_a^* + x_2) = 168 + 2 \cdot 2 \cdot (1 - 0{,}1) = 171{,}6\,mm$

$$a_w = a = \frac{d_1 + d_2}{2} = \frac{208}{2} = 104\,mm$$

(2) <u>pero</u> -> $d = 40$ mm

$M_k = 314$ Nm

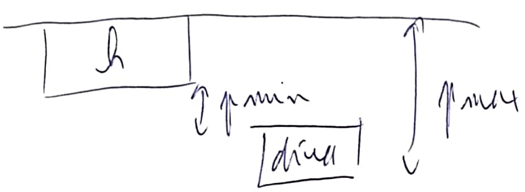
pero ... $12 \times 8 \times 50$

$$p = \frac{F}{S_k} \quad \to F = \frac{314000}{20} = 15\,700 \text{ N}$$

$$\to S_k = \frac{8}{2} \cdot (50 - 12) = 152 \text{ mm}^2 \quad \to p = \frac{15\,700}{152} = \underline{\underline{103{,}3 \text{ MPa}}}$$

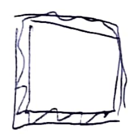
$$\tau = \frac{F}{S_\tau} \qquad S_\tau = (50-12)\cdot 12 = 456 \text{ mm}^2 \to \tau = \frac{15\,700}{456} = \underline{\underline{34{,}4 \text{ MPa}}}$$

(3) <u>uložení</u> -> přesah -> jednotná hřídel



<u>PP2</u> A) D: $l = 0{,}2$ m, $c = 30$ mm, $h = 2$ mm, $M = 1{,}8 \cdot 10^5$ Nmm

U: τ_{MAX}



$$\tau_{MAX} = \frac{M_K}{W_K} = \frac{1{,}8 \cdot 10^5}{3600} = \underline{\underline{50 \text{ N/mm}^2}}$$

$$W_K = 2 \cdot c^2 \cdot h = 2 \cdot 30^2 \cdot 2 = 3600$$

B) D: $l = 0{,}7$ m, $c = 40$ mm, $h = 2$ mm, $M = 3232$ Nmm, $G = 0{,}81 \cdot 10^5$ N/mm²

U: φ_{A-B}

$$\varphi_{A-B} = \frac{M_K \cdot l}{G \cdot J_K} = \frac{3232 \cdot 0{,}7}{0{,}81 \cdot 10^5 \cdot 320} = 0{,}0873 \text{ rad} \approx \underline{\underline{5^\circ}}$$

$$J_K = \frac{1}{3} \cdot 3 \cdot c \cdot h^3 = 320 \text{ mm}^4$$

MATEMATIKA: 19/20

① $f(x) = \ln(2x-1) - \frac{x}{2}$ $\longrightarrow$ $f(x_0) = \ln(1) - \frac{1}{2} = -\frac{1}{2}$

a) $D(f)$, f', $T_2(x)$, $x_0 = 1$, $f(x)$ pur $x = 5/4$

$$2x - 1 > 0$$
$$2x > 1$$
$$x > \frac{1}{2} \rightarrow D(f) = \left(\frac{1}{2}; \infty\right) = D(f')$$

$$f'(x) = \frac{2}{2x-1} - \frac{1}{2} \qquad f'(x_0) = \frac{2}{2-1} - \frac{1}{2} = 2 - \frac{1}{2} = \frac{3}{2}$$

$$f''(x) = -2 \cdot (2x-1)^{-2} \cdot 2 - 0 = \frac{-4}{(2x-1)^2} \rightarrow f''(x_0) = \frac{-4}{1} = -4$$

$$T_2(x) = -\frac{1}{2} + \frac{\frac{3}{2}}{1!} \cdot (x-1) + \frac{-4}{2!} \cdot (x-1)^2$$

$$\boxed{T_2(x) = -\frac{1}{2} + \frac{3}{2}(x-1) - 2(x-1)^2}$$

$$T_2\left(\frac{5}{4}\right) = -\frac{1}{2} + \frac{3}{2} \cdot \left(\frac{5}{4} - 1\right) - 2\left(\frac{5}{4} - 1\right)^2 = -\frac{1}{2} + \frac{3}{2} \cdot \frac{1}{4} - 2 \cdot \frac{1}{16}$$
$$= -\frac{1}{2} + \frac{3}{8} - \frac{2}{16} = \frac{-8+6-2}{16} = -\frac{4}{16} = -\frac{1}{4}$$

$$\Rightarrow \underline{f(5/4) \doteq -\frac{1}{4}}$$

b) $R_3(x)$; $R_3(\frac{5}{4})$

$$R_3(x) = \frac{f'''(\xi)}{3!} \cdot (x - x_0)^3 = \frac{\frac{16}{(2\xi-1)^3}}{3!} \cdot (x-1)^3 = \frac{8}{3} \cdot (2\xi - 1)^{-3} \cdot (x-1)^3$$

$$f'''(x) = -4 \cdot (2x-1)^{-3} \cdot (-2) \cdot 2 = \frac{16}{(2x-1)^3}$$

$$\left|R_3\left(\frac{5}{4}\right)\right| \leq \frac{8}{3} \cdot (2 \cdot 1 - 1)^{-3} \cdot \left(\frac{5}{4} - 1\right)^3 = \frac{\cancel{8}^{1}}{3} \cdot \frac{1}{\cancel{64}_{8}} = \frac{1}{24}$$

$$\underline{\left|R_3\left(\frac{5}{4}\right)\right| \leq \frac{1}{24}}$$

② $f(x) = x^2 \cdot e^{x-2}$ → $y_0 = 4e^0 = 4$

a) t: $x_0 = 2$

t: $y = f'(x_0)\cdot(x - x_0) + y_0$

$f'(x) = 2x \cdot e^{x-2} + x^2 \cdot e^{x-2}$ → $f'(x_0) = 2\cdot 2\cdot e^0 + 2^2 \cdot e^0$

$f'(x_0) = 4 + 4 = 8$

t: $y = 8\cdot(x-2) + 4$

$y = 8x - 12$

b) extrémy $f(x;y) = x + 4\sqrt{x} - 2y$ na $y = x - 1$; $x \in \langle 0; 9\rangle$

→ dosadit → $\cancel{\ldots}x + 4\cdot\sqrt{x} - 2x + 2$

$\cancel{f(0) = 0 + 4\sqrt{0} - 2\cdot 0 + 2 = 2 \to 2 = x - 1 \to x_1 = 3}$

$\cancel{g(9) = 9 + 4\cdot 3 - 18 + 2 = 5 \to 5 = x - 1 \to x_2 = 6}$

$x_1 = 0 \to y_1 = -1$

$g'(x) = 1 + 4\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{x}} - 2 = \frac{2}{\sqrt{x}} - 1$ $\quad x_2 = 9 \to y_2 = 8$

$g'(x) = 0$

⇓

$\frac{2}{\sqrt{x}} - 1 = 0$

$\frac{2}{\sqrt{x}} = 1$

$\sqrt{x} = 2$

$x_3 = 4$

$y = 4 - 1$

$y = 3$

$\cancel{1)\ f(3;2) = 3 + 4\sqrt{3} - 2\cdot 2 = 4\sqrt{3} - 1 \doteq 5{,}9}$

$\cancel{2)\ f(6;5) = 6 + 4\sqrt{5} - 2\cdot 5 = 4\sqrt{5} - 4 \doteq 7}$

3) $f(4;3) = 4 + 4\cdot\sqrt{4} - 2\cdot 3 = \underline{6}$

1) $f(0;-1) = 0 + 4\cdot 0 - 2\cdot(-1) = \underline{2}$

2) $f(9;8) = 9 + 4\sqrt{9} - 2\cdot 8 = \underline{5}$

max $f = f(4;3) = 6$; min $f = f(0;-1) = 2$

②

③ $\ddot{x} - 5\dot{x} + 6x = 0$

a) FS ; ORˡ , Cauchy $x(0) = 2$; $\dot{x}(0) = 1$

$\lambda^2 - 5\lambda + 6 = 0$

$\lambda_1 = 2 \quad , \quad \lambda_2 = 3$

FS: $\{ \varphi_1(t) = e^{2t} ; \varphi_2(t) = e^{3t} \}$ ✓

ORˡ: $x(t) = c_1 e^{2t} + c_2 e^{3t}$ ✓

Cauchy: $\dot{x}(t) = 2c_1 e^{2t} + 3c_2 e^{3t}$

$2 = c_1 e^0 + c_2 e^0 \rightarrow 2 = c_1 + c_2 \rightarrow c_1 = 2 - c_2$

$1 = 2c_1 e^0 + 3c_2 e^0 \rightarrow 1 = 2c_1 + 3c_2$

$1 = 4 - 2c_2 + 3c_2$

$c_2 = -3 \rightarrow c_1 = 5$

$x(t) = 5e^{2t} - 3e^{3t} \quad . \ t \in (-\infty ; \infty)$

b) $\ddot{x} + 4x = 0 \rightarrow$ FS ; part. řešení 1) $f(t) = 2t e^{3t}$; 2) $f(t) = 3 \sin t$

$\lambda^2 + 4 = 0 \rightarrow$ FS: $\{ \varphi_1(t) = \cos(2t) ; \varphi_2(t) = \sin(2t) \}$

$\lambda^2 = -4$

$\lambda = \pm\sqrt{-4}$

$\lambda = \pm 2i$

1) $f(t) = 2t e^{3t} \rightarrow$ polynom 1. stupně

$\alpha = 3 ; \beta = 0 \rightarrow \omega = 3 \neq \lambda_1 \neq \lambda_2 \rightarrow s = 0$

~~$x(p) = e^{3t}(A \cos 0t + B \sin 0t) \cdot t^0 = A e^{3t}$~~

~~$\dot{x}(p) = 3A e^{3t}$~~

~~$\ddot{x}(p) = 9A e^{3t} \Rightarrow 9Ae^{3t} + 4Ae^{3t} = 2te^{3t}$~~

1) $f(t) = 2t e^{3t}$ → polynom 1. stupně

$\alpha = 3;\ \beta = 0 \to \omega = 3 \neq \lambda_1 \neq \lambda_2 \to k = 0$

$x(p) = e^{3t} \cdot \left((A_1 t + A_0)\cos(0) + \cancel{(B_1 t + B_0)\sin(0)} \right) \cdot \cancel{t^0}$

$x(p) = e^{3t} \cdot (A_1 t + A_0) = A_1 t e^{3t} + A_0 e^{3t}$

$\dot{x}(p) = A_1 \cdot (e^{3t} + 3t e^{3t}) + 3A_0 e^{3t} = A_1 e^{3t} + 3A_1 t e^{3t} + 3A_0 e^{3t}$

$\ddot{x}(p) = 3A_1 e^{3t} + 3A_1 \cdot (e^{3t} + 3t e^{3t}) + 9A_0 e^{3t}$

$= 6A_1 e^{3t} + 9A_1 t e^{3t} + 9A_0 e^{3t}$

$6A_1 e^{3t} + \underline{9A_1 t e^{3t}} + 9A_0 e^{3t} + \underline{4A_1 t e^{3t}} + 4A_0 e^{3t} = 2t e^{3t}$

$6A_1 e^{3t} + 13A_1 t e^{3t} + 13A_0 e^{3t} = 2t e^{3t}$

$6A_1 + 13A_1 t + 13A_0 = 2t$

AMA → stačí takto

$x(p) = A_1 t e^{3t} + A_0 e^{3t} \to \underline{x(p) = (At + B) \cdot e^{3t}}$

2) $f(t) = 3\sin 2t$ → pol. 0. stupně

$\alpha = 0;\ \beta = 2 \to \omega = 2i = \lambda_1 \neq \lambda_2 \to k = 1$

$x(p) = e^{0t} \cdot (A \cos 2t + B \sin 2t) \cdot t$

$\underline{x(p) = (A \cos 2t + B \sin 2t) \cdot t}$

TE1 - 10 jednoduchých otázek a, b, c

TE2 - počítání CNC

$v_c = 126$ m/min
$f = 0{,}4$
$a_p = 2$
$R_m = 700$
$D = 80$
$L = 200$

U: a) n
b) A_D
c) F_c
d) P_c
e) t_{AS}

a) $n = \frac{1000 \cdot v_c}{\pi \cdot D} = 501 \ \text{min}^{-1}$

b) $A_D = 0{,}4 \cdot 2 = 0{,}8 \ \text{mm}^2$

c) $F_c = k_c \cdot A_D = c_k \cdot R_m \cdot A_D = 4 \cdot 700 \cdot 0{,}8 = \underline{\underline{2240\ N}}$

d) $P_c = F_c \cdot \frac{v_c}{60} = 2240 \cdot \frac{126}{60} = \underline{\underline{4{,}7\ kW}}$

e) $t_{AS} = \frac{60 \cdot L}{f \cdot n} = \frac{60 \cdot 200}{0{,}4 \cdot 501} = \underline{\underline{60\ s}}$

NOM - 10 otázek a, b, c, d

ČNS - varianta A

1) ozubení $\rightarrow m = 2$ mm; $i = 3{,}79$; $z_1 = 19$; $x_1 = +0{,}1$; $x_2 = -0{,}1$

U: z_2, d_1, d_2, d_{a1}, a

$z_2 = z_1 \cdot i = 19 \cdot 3{,}79 = \underline{\underline{72}}$

$d_1 = m \cdot z_1 = 2 \cdot 19 = \underline{\underline{38\ mm}}$

$d_2 = m \cdot z_2 = \underline{\underline{144}}$ mm

$d_{a1} = d_1 + 2m\,(h_a^* + x_1) = 38 + 2 \cdot 2 \cdot (1 + 0{,}1) = \underline{\underline{42{,}4\ mm}}$

$a = \frac{d_1 + d_2}{2} = \underline{\underline{91\ mm}}$

② pero

$d = 28\ mm$ / b / h / l

pero ... $8 \times 7 \times 50$ (šířka 8mm)

$M_k = 150\ Nm$

$l_a = l - b$

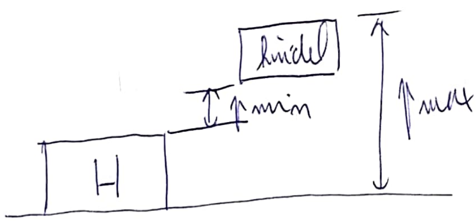
a) kontaktní tlak na boku pera

- kontaktní plocha na boku pera $\boxed{S_K = \frac{h}{2}\cdot l_a} = 3{,}5 \cdot 42 = 147\ mm^2$
- přenášená síla $\boxed{F = \frac{M_k}{r}} = \frac{150000}{14} = 10\,714\ N$
- kontaktní tlak $\boxed{p = \frac{F}{S_K}} = \frac{10714}{147} = 72{,}88\ MPa$
- střižná plocha pera $\boxed{S_S = l_a \cdot b} = 42 \cdot 8 = 336\ mm^2$
- smykové napětí $\boxed{\tau = \frac{F}{S_S}} = \frac{10714}{336} = 31{,}89\ MPa$

$\boxed{p = \frac{4M_k}{d_H \cdot h \cdot l_a}}$ $\boxed{\tau = \frac{2M_k}{d_H \cdot b \cdot l_a}}$

③ uložení

-> toleranční pole -> hřídel do díry s přesahem -> vyznačit největší a nejm. přesah -> soustava jednotné díry



varianta B

① ozubení: $m=2$; $i=3,62$; $z_1=21$, $x_1=+0,1$; $x_2=-0,1$

U: z_2^*, d_1, d_2, d_{a1}, a

$z_2 = z_1 \cdot i = \underline{76}$

$d_1 = m \cdot z_1 = \underline{42\,mm}$

$d_2 = m \cdot z_2 = \underline{152\,mm}$

$d_{a1} = d_1 + 2m \cdot (h_a^* + x_1) = 42 + 2 \cdot 2 \cdot (1 + 0,1) = \underline{46,4\,mm}$

$a = \frac{d_1 + d_2}{2} = \frac{194}{2} = \underline{97\,mm}$

② pero -> $d = 32\,mm$

pero ... $10 \times 8 \times 50$ (šířka 10)

$M_k = 225\,Nm$

a) $p_k = \frac{F}{S_k}$ -> $S_k = \frac{h}{2} \cdot (l - b) = 4 \cdot 40 = 160\,mm^2$

$F = \frac{M_k}{r} = \frac{225000}{16} = 14\,062,5\,N$

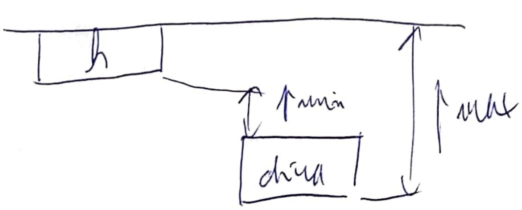
=> $p_k = \frac{14062,5}{160} = \underline{87,9\,MPa}$

[illegible]

b) $\tau = ?$ -> $S_\tau = l_a \cdot b = (l - b) \cdot b = 40 \cdot 10 = 400\,mm^2$

$\tau = \frac{F}{S_\tau} = \frac{14\,062,5}{320} = \cancel{35,2\,MPa}\ \underline{43,95\,MPa}$

③ uložení – přesah, jednotná hřídel



PP2

D: $l = 0{,}5\,m$, $c = 30\,mm$, $h = 3\,mm$, $M = 1 \cdot 10^4\,Nmm$, $G = 0{,}8 \cdot 10^5\,N/mm^2$

U: φ_{A-B}

$$\varphi_{A-B} = \frac{M \cdot l}{G \cdot J} = \frac{1 \cdot 10^4 \cdot \cancel{0{,}5}\ 500}{0{,}8 \cdot 10^5 \cdot 1080} = 0{,}0579 \approx 3{,}3^\circ$$

?

$$J = \cancel{2 \cdot c^2 \cdot h} \ \frac{1}{3} \cdot 4 \cdot \underset{c}{30} \cdot \underset{h}{3^2} = 1080$$

2

MATEMATIKA: 20/21

$$\det(A-\lambda E)=0$$

(1) a) vlastní čísla

$$A=\begin{pmatrix}2&2&-1\\0&1&0\\5&3&4\end{pmatrix} \rightarrow \det(A-\lambda E)=\begin{vmatrix}2-\lambda&2&-1\\0&1-\lambda&0\\5&3&4-\lambda\end{vmatrix}=$$

$$\cancel{=(2-\lambda)\cdot(1-\lambda)\cdot(4-\lambda)+0+0-(5\lambda-5)-0-0=}$$

$$\cancel{=(2-3\lambda+\lambda^2)\cdot(4-\lambda)-5\lambda+5=0}$$

$$\cancel{(2-3\lambda+\lambda^2)\cdot(4-\lambda)=5\lambda+5}$$

$$\cancel{\lambda^2-3\lambda+2=0} \qquad \cancel{\lambda_3=4} \qquad \cancel{\lambda_4=1}$$

$$\lambda_1=1\ ;\ \lambda_2=2$$

$$=(2-\lambda)\cdot(1-\lambda)\cdot(4-\lambda)+0+0+5\cdot(1-\lambda)+0+0 \neq$$

$$(8-6\lambda+\lambda^2)\cdot(1-\lambda)+5\cdot(1-\lambda)=0$$

$$(\lambda^2-6\lambda+13)\cdot(1-\lambda)=0$$

$$\rightarrow \lambda_1=1$$

$$D=36-4\cdot1\cdot13=-16=16i^2$$

$$\cancel{\lambda_{2,3}}\qquad \lambda_{2,3}=\frac{6\pm\sqrt{16i^2}}{2}=\begin{cases}3+2i\\3-2i\end{cases}$$

b) $\lambda_1=1$ $$\begin{pmatrix}1&2&-1\\0&0&0\\5&3&3\end{pmatrix}\sim\begin{pmatrix}1&2&-1\\0&0&0\\8&9&0\end{pmatrix}\rightarrow 8x+9y=0$$

$$8x=-9y$$

$$x=-9;\ y=8$$

$$x+2y-z=0 \qquad z=7$$
$$-9+16-z=0$$

$$u=(-9t;\ 8t;\ 7t)^T\ ;\ t\in\mathbb{R}\ ;\ t\neq0$$

(1)

③ $f(x) = \ln(x-1) - \sqrt{x-1}$ $\longrightarrow$ $y_0 = \ln(2-1) - \sqrt{2-1}$

a) $D(f)$, $t: x_0 = 2$, odhad v $x = \frac{5}{2}$

$y_0 = 0 - 1$

$y_0 = -1$

$x - 1 > 0 \quad \wedge \quad x - 1 \geq 0 \qquad D(f) = (1; \infty)$

$x > 1 \qquad\qquad x \geq \cancel{0}\,1$

$t: y = f'(x_0) \cdot (x - x_0) + y_0$

$f'(x) = \frac{1}{x-1} - \frac{1}{2}(x-1)^{-\frac{1}{2}} = \frac{1}{x-1} - \frac{1}{2\sqrt{x-1}} \rightarrow f'(x_0) = 1 - \frac{1}{2} = \frac{1}{2}$

$t: y = \frac{1}{2} \cdot (x-2) \cancel{+} - 1$

$\underline{y = \frac{x}{2} - 2} \rightarrow f\left(\frac{5}{2}\right) = \frac{\frac{5}{2}}{2} - 2 = \frac{5}{4} - \frac{8}{4} = \underline{-\frac{3}{4}}$

b) extrémy $f(x,y) = 3\sqrt{x} - y$ na úsečce $y = x + 1$; $x \in \langle 0; 4 \rangle$

$\rightarrow$ dosadím úsečku do fce: $f(x) = 3\sqrt{x} - x - 1 = g(x)$

$\cancel{g(0) = 0 - 0 - 1 = -1 \rightarrow -1 = x + 1 \rightarrow x = -2}$

$\cancel{g(4) = 6 - 4 - 1 = 1 \rightarrow 1 = x + 1 \rightarrow x = 0}$

$x_1 = 0 \rightarrow y_1 = 1$

$x_2 = 4 \rightarrow y_2 = 5$

$g'(x) = 3 \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}} - 1 = \frac{3}{2} \frac{1}{\sqrt{x}} - 1 = \frac{3}{2\sqrt{x}} - 1$

$g'(x) = 0$

$\Downarrow$

$\frac{3}{2\sqrt{x}} - 1 = 0$

$\frac{3}{2\sqrt{x}} = 1$

$3 = 2\sqrt{x}$

$\sqrt{x} = \frac{3}{2}$

$x = \frac{9}{4} \rightarrow y = \frac{9}{4} + \frac{4}{4} = \frac{13}{4}$

②

$\cancel{f(-2;-1) = \dots}$ nebo $f(4;5) = 3 \cdot 2 - 5 = 1$

$\boxed{f(0;1) = -1 \qquad f\left(\frac{9}{4}; \frac{13}{4}\right) = \frac{9}{2} - \frac{13}{4} = \frac{5}{4}}$

$\underline{\max f = f\left(\frac{9}{4}; \frac{13}{4}\right) = \frac{5}{4}}$

$\underline{\min f = f(0;1) = -1}$

(3) $\ddot{x} - 3\dot{x} - 4x = 6e^{-2t}$

a) $\ddot{x} - 3\dot{x} - 4x = 0$

$\lambda^2 - 3\lambda - 4 = 0$

$\lambda_1 = -1 \quad \lambda_2 = 4$

FS: $\{\varphi_1(t) = e^{-t} ; \varphi_2(t) = e^{4t}\}$

OR: $x(t) = c_1 e^{-t} + c_2 e^{4t} ; t \in (-\infty; \infty)$

$\dot{x}(t) = -c_1 e^{-t} + 4c_2 e^{4t}$

b) $f(t) = 6e^{-2t}$ -> polynom 0. stupně

$\alpha = -2 ; \beta = 0 \rightarrow \omega = -2 \rightarrow k = 0$

$x(p) = e^{-2t} \cdot (A\cos 0t + \cancel{B\sin 0t}) \cdot \cancel{t^0}$

$\boxed{x(p) = Ae^{-2t}}$

$\dot{x}(p) = -2Ae^{-2t}$

$\ddot{x}(p) = 4Ae^{-2t}$

$\cancel{4Ae^{-2t}} + 6Ae^{-2t} - \cancel{4Ae^{-2t}} = 6e^{-2t}$

$6A = 6$

$A = 1$

$\boxed{x(p) = e^{-2t}}$

ORNR: $x(t) = c_1 e^{-t} + c_2 e^{4t} + e^{-2t} ; t \in (-\infty; \infty)$

c) Cauchy: $x(0) = 1 ; \dot{x}(0) = 3$

$1 = c_1 e^0 + c_2 e^0 \rightarrow 1 = c_1 + c_2 \rightarrow$

$3 = -c_1 e^0 + 4c_2 e^0 \rightarrow 3 = -c_1 + 4c_2 \rightarrow c_1 = 4c_2 - 3$

$1 = 4c_2 - 3 + c_2$

$4 = 5c_2$

$c_2 = \frac{4}{5}$

$c_1 = 4 \cdot \frac{4}{5} - 3$

$c_1 = \frac{1}{5}$

$\boxed{x(p) = \frac{1}{5}e^{-t} + \frac{4}{5}e^{4t} + e^{-2t} ; t \in (-\infty; \infty)}$?

c) $x(t) = c_1 e^{-t} + c_2 e^{4t} + e^{-2t}$

$\dot{x}(t) = -c_1 e^{-t} + 4c_2 e^{4t} - 2e^{-2t}$

$1 = c_1 e^{0} + c_2 e^{0} + e^{0} \rightarrow \cancel{1} = c_1 + c_2 + \cancel{1} \rightarrow c_1 = -c_2$

$3 = -c_1 e^{0} + 4c_2 e^{0} - 2e^{0} \rightarrow 3 = -c_1 + 4c_2 - 2$

$3 = 5c_2 - 2$

$c_2 = 1 \rightarrow c_1 = -1$

$x(t) = -e^{-t} + e^{4t} + e^{-2t} \; ; \; t \in \mathbb{R}$

TE1 - 10 jednoduchých otázek a, b, c

TE2 - podélné soustružení

Z: $v_c = 200$ m/min

posuv $f = 0{,}5$ mm/ot

hloubka $a_p = 4$ mm

$R_m = 750$ N/mm^2

$D = 60$ mm

$L = 120$ mm

U: a) $n = ?$ min^{-1}

b) $A_D = ?$ mm^2

c) $F_c = ?$ N

d) $V = ?$ cm^3/min

e) $Q = ?$ kJ

řezná rychlost: $v_c = \frac{\pi \cdot D \cdot n}{1000}$ [m/min]

otáčky: $n = \frac{1000 \cdot v_c}{\pi \cdot D}$ [min^{-1}] ← m/min, ↑ mm

průřez třísky: $A_D = f \cdot a_p$ [mm^2] (posuv, hloubka)

řezná složka síly: $F_c = k_c \cdot A_D$ [N]

měrná ř. síla $k_c = C_k \cdot R_m$ (3 až 5)

objem materiálu: $V = A_D \cdot v_c$ [cm^3/min]

řezný výkon: $P_c = F_c \cdot \frac{v_c}{60}$ [W]

strojní čas: $t_{AS} = \frac{60 \cdot L}{f \cdot n}$ [s] (L – mm, f – mm/ot, n – min^{-1})

práce: $Q = P_c \cdot t_{AS}$ [J]

a) $n = \frac{1000 \cdot v_c}{\pi \cdot D} = \frac{1000 \cdot 200}{\pi \cdot 60} = \underline{\underline{1061 \text{ min}^{-1}}}$

b) $A_D = f \cdot a_p = 0{,}5 \cdot 4 = \underline{\underline{2 \text{ mm}^2}}$

c) $F_c = k_c \cdot A_D = C_k \cdot R_m \cdot A_D = 4 \cdot 750 \cdot 2 = \underline{\underline{6000 \text{ N}}}$

d) $V = A_D \cdot v_c = 2 \cdot 200 = \underline{\underline{400 \text{ cm}^3/\text{min}}}$

e) $Q = P_c \cdot t_{AS} = F_c \cdot \frac{v_c}{60} \cdot \frac{60 \cdot L}{f \cdot n} = 6000 \cdot \frac{200}{\cancel{60}} \cdot \frac{\cancel{60} \cdot 120}{0{,}5 \cdot 1061} = \underline{\underline{271 \text{ kJ}}}$

ČMS

ozubení

- roztečná kružnice: $d = m \cdot z$
- hlavová k. : $d_a = d + 2m\,(h_a^* + x)$
- patní k. : $d_f = d_2 - 2m\,(h_a^* + c^*) + 2xm$
- valivá k. : $d_w = d \cdot \frac{\cos\alpha}{\cos\alpha_w}$
- základní k. : $d_b = d \cdot \cos\alpha$

$\longrightarrow h_a^* = 1 \ ; \ c^* = 0{,}25 \ ; \ \alpha = 20^\circ$

- $z_2 = z_1 \cdot i$
- osová vzdál. osová: $a = \frac{1}{2} m\,(z_1 + z_2) = \frac{d_1 + d_2}{2}$
- o. v. valivá $d_w = a$

D: $m = 2\,mm$
$z_1 = 19$
$x_1 = x_2 = 0$

U: 1) z_2 ; $i = 3{,}99 \rightarrow z_2 = z_1 \cdot i = 19 \cdot 3{,}99 = \underline{\underline{76}}$

2) $d_1 = m \cdot z_1 = \underline{\underline{38}}$

3) $d_{a_1} = d_1 + 2m\,(h_a^* + x_1) = 38 + 2 \cdot 2 \cdot (1 + 0) = \underline{\underline{42}}$

$d_{f1} = d_1 - 2m\,(h_a^* + c^*) + 2x_1 m = 38 - 2 \cdot 2 \cdot (1 + 0{,}25) + 0 = \underline{\underline{33}}$

4) $a = \frac{1}{2} m\,(z_1 + z_2) = \cancel{\frac{d_1 + d_2}{2}} \ \frac{1}{2} \cdot 2 \cdot (19 + 76) = \underline{\underline{95}}$

pero -> $d = 32\ mm$

$M_k = 160\ Nm$

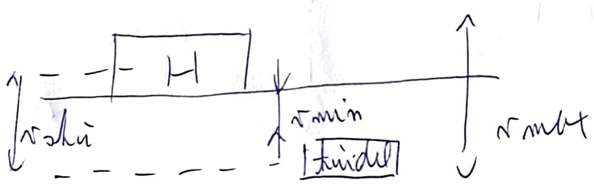
pero ... $10 \times 8 \times 50$

1) $F = \frac{160000}{16} = \underline{\underline{10\,000\ N}}$

2) $S_K = \frac{8}{2} \cdot (50-10) = 160\ mm^2 \rightarrow p = \frac{F}{S_K} = \frac{10000}{160} = \underline{\underline{62{,}5\ MPa}}$

3) $S_\tau = (50-10) \cdot 10 = 400\ mm^2 \rightarrow \tau = \frac{10000}{400} = \underline{\underline{25\ MPa}}$

uložení -> s vůlí -> [illegible] díra



PP

- vzpěr 3. druhu -> $n = 2$; kruh => $J_{min} = \frac{\pi d^4}{64}$

D: $\sigma_K = R_e = 260\ N/mm^2$, $\sigma_u = R_u = 200\ N/mm^2$, $E = 2 \cdot 10^5\ N/mm^2$, $l = 600\ mm$, $d = 12\ mm$, $k_E = 3$

① F_{KR}, σ_{KR} ② F_D, σ_D

$$F_{KR} = \frac{n \cdot \pi^2 \cdot E \cdot J_{min}}{l^2} = \frac{n \cdot \pi^3 \cdot E \cdot d^4}{64\, l^2} = \frac{2 \cdot \pi^3 \cdot 2 \cdot 10^5 \cdot 12^4}{64 \cdot 600^2} = \underline{\underline{11162{,}26\ N}}$$

$$\sigma_{KR} = \frac{F_{KR}}{A} = \frac{11162{,}26}{\frac{\pi \cdot 12^2}{4}} = \underline{\underline{98{,}7\ MPa}}$$ $< \sigma_u$ => Euler ✓

$$F_{KRD} = \frac{F_{KR}}{k_E} = \frac{11162{,}26}{3} = \underline{\underline{3720{,}25}}$$

$$\sigma_D = \frac{F_D}{A} = \underline{\underline{32{,}9\ MPa}}$$

② D $h = 130\,mm$, $l = 1200\,mm$, $t = 4\,mm$, $R \approx 15\,mm$, $G = 0{,}8 \cdot 10^5\,N/mm^2$,

$M_K = 2 \cdot 10^4\,Nmm$, $(a = 25\,m, b = 33\,m)$

U: τ_{max}, φ_{A-B}

$$\tau_{MAX} = \frac{M_K}{W_K} = \frac{M_K}{\frac{1}{3} h t^2} = \frac{2 \cdot 10^4}{\frac{1}{3} \cdot 130 \cdot 4^2} = \underline{\underline{28{,}85\,N/mm^2}}$$

$$\varphi_{A-B} = \frac{M \cdot l}{G \cdot J} = \frac{M \cdot l}{G \cdot \frac{1}{3} \cdot h t^3} = \frac{2 \cdot 10^4 \cdot 1200}{0{,}8 \cdot 10^5 \cdot \frac{1}{3} \cdot 130 \cdot 4^3} = 0{,}108\,rad \approx \underline{\underline{6{,}}}$$

MECH

Lagrange $\rightarrow$ 1) $$\boxed{\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}}\right) - \frac{\partial E_k}{\partial q} = Q}$$

E_k ... kin. energie
t ... čas
q ... zobecněná souř
$\dot{q}$ ... zobec. rychlos
Q ... z. síla

2) $$\boxed{E_k = \frac{1}{2} I_{20} \omega_2^2 + \frac{1}{2} m_3 v_3^2 + \frac{1}{2} m_4 v_4^2}$$

MATEMATIKA: 21/22

(1) $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 4 & -2 & 5 \end{pmatrix}$; $\lambda_1 = 1$

$\begin{pmatrix} 0 & 0 & 0 \\ 2 & 2 & 1 \\ 4 & -2 & 4 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 \\ 2 & 2 & 1 \\ 6 & 0 & 5 \end{pmatrix}$ $\begin{array}{l} \to 2x+2y+z=0 \to 2\cdot(-5)+2y+6=0 \\ \to 6x+5z=0 \end{array}$

$-10+2y+6=0$

$2y=4$

$y=2$

$6x=-5z$

$x=-5$

$z=6$

$u = (-5t; 2t; 6t)^T$; $t \in \mathbb{R}$; $t \neq 0$

(2) $f(x) = x^3 \cdot e^{x-1} \to y_0 = 1^3 \cdot e^{1-1} = 1$

$\to D(f)$; t v $x_0 = 1$; $f(x)$ v $x = \frac{4}{3}$ pomocí tečny

- $D(f) = \mathbb{R}$
- t: $y = f'(x_0) \cdot (x - x_0) + y_0$

$f'(x) = 3x^2 \cdot e^{x-1} + x^3 \cdot e^{x-1} \to f'(x_0) = 3 \cdot 1^2 \cdot e^{1-1} + 1^3 \cdot e^{1-1}$

$= 3 + 1 = 4$

t: $y = 4 \cdot (x-1) + 1$

$f\left(\frac{4}{3}\right) \doteq 4 \cdot \left(\frac{4}{3} - 1\right) + 1 = \frac{16}{3} - 4 + 1 = \frac{16-9}{3} = \frac{7}{3}$

(3) $\ddot{x} + 2\dot{x} + 5x = 2\sin(2t)$

$$\lambda^2 + 2\lambda + 5 = 0$$

$$D = 4 - 4\cdot 1\cdot 5 = -16 = 16i^2$$

$$\lambda_{1,2} = \frac{-2 \pm \sqrt{16i^2}}{2} = \begin{cases} -1 + 2i \\ -1 - 2i \end{cases}$$

kořeny	FS	OŘ
λ_1, λ_2 reálné různé	$e^{\lambda_1 t}$; $e^{\lambda_2 t}$	$c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$
dvojnás. kořen $\lambda_1 = \lambda_2$	$e^{\lambda_1 t}$; $t e^{\lambda_1 t}$	$c_1 e^{\lambda_1 t} + c_2 t e^{\lambda_1 t}$
$\lambda_1 = 0$; λ_2	1; $e^{\lambda_2 t}$	$c_1 + c_2 e^{\lambda_2 t}$
$\lambda_{1,2} = \pm i\omega$	$\cos\omega t$; $\sin\omega t$	$c_1 \cos\omega t + c_2 \sin\omega t$
$\lambda_{1,2} = \alpha \pm i\omega$	$e^{\alpha t}\cos\omega t$; $e^{\alpha t}\sin\omega t$	$e^{\alpha t} c_1 \cos\omega t + e^{\alpha t} c_2 \sin\omega t$

$$FS = \{ \varphi_1(t) = e^{-t}\cos 2t;\ \varphi_2(t) = e^{-t}\sin 2t \}$$

(4) vekt. pole $\vec{f} = (-y; x; z)$ → znaménko ⊖

→ div $\vec{f}$; tok domčen tělesem $M = \{[x,y,z] \in E_3;\ x^2+y^2+4 \le z \le 8\}$

• $\operatorname{div} \vec{f} = \frac{\partial \vec{f}}{\partial x} + \frac{\partial \vec{f}}{\partial y} + \frac{\partial \vec{f}}{\partial z} = 0 + 0 + 1 = \underline{\underline{1}}$

• G-Ostrogr.: $\iint_S \vec{f}\cdot d\vec{p} = \iiint_M \operatorname{div}\vec{f}\, dx\,dy\,dz = -\iiint 1\, dx\,dy\,dz$

↓

musím převést ~~[illegible]~~ substituci:

$$x^2+y^2+4 \le z \le 8 \longrightarrow 0 \le r \le 2 \leftarrow \sqrt{4}$$

$$0 \le u \le 2\pi \leftarrow 360^\circ$$

př.:

$x = r\cos(u)$

$y = r\sin(u)$

$J = r \leftarrow$ Jakobián

počítání $\longrightarrow$ $$= -\int_0^2\int_0^{2\pi}\int_{x^2+y^2+4}^{8} 1\cdot \overset{J}{r}\, dz\, du\, dr = -\int_0^2\int_0^{2\pi} r\cdot [z]_{x^2+y^2+4}^{8}\, du\, dr =$$

$$= -\int_0^2 r\int_0^{2\pi}(8-x^2-y^2-4)\, du\, dr = -\int_0^2 r\int_0^{2\pi}(8-r^2\cos^2(u)-r^2\sin^2(u)-4)\, du\, dr =$$

$$= -\int_0^2 r\int_0^{2\pi}(8-r^2-4)\, du\, dr = -\int_0^2 (4r-r^3)\cdot [u]_0^{2\pi}\, dr = -2\pi\cdot\int_0^2 (4r-r^3)\, dr =$$

$$= -2\pi\cdot\left[2r^2-\frac{r^4}{4}\right]_0^2 = -2\pi\cdot((8-4)-0) = \underline{\underline{-8\pi}}$$

(5) $\dot{x} = x - \frac{1}{y} \longrightarrow y \neq 0$

$\dot{y} = 2x - y + 1 \qquad G = \{[x,y] \in E_2 \,;\, y \neq 0\} \longrightarrow$

$\longrightarrow G_1 = \{[x,y] \in E_2 \,;\, y > 0\};\ G_2 = \{[x,y] \in E_2 \,;\, y < 0\}$

(6) $f(x) = x - \frac{1}{\sqrt{x}} \longrightarrow f(x_0) = 1 - \frac{1}{\sqrt{1}} = 0 \qquad f(x) = x - x^{-\frac{1}{2}}$

$\rightarrow T_2(x)$; $x_0 = 1$; $R_3(x)$; $f(x)$ pro $x = \frac{4}{3}$

$$T_2(x) = f(x_0) + \frac{f'(x_0)}{1!} \cdot (x - x_0)^1 + \frac{f''(x_0)}{2!} \cdot (x - x_0)^2$$

$$f'(x) = 1 + \frac{1}{2} x^{-\frac{3}{2}} = 1 + \frac{1}{2\sqrt{x^3}} \rightarrow f'(x_0) = 1 + \frac{1}{2} = \frac{3}{2}$$

$$f''(x) = 0 - \frac{1}{2} \cdot \frac{3}{2} x^{-\frac{5}{2}} = -\frac{3}{4\sqrt{x^5}} \rightarrow f''(x_0) = -\frac{3}{4}$$

$$f'''(x) = -\frac{3}{4} \cdot \left(-\frac{5}{2}\right) \cdot x^{-\frac{7}{2}} = \frac{15}{8\sqrt{x^7}} \rightarrow f'''(x_0) = \frac{15}{8}$$

$$T_2(x) = 0 + \frac{\frac{3}{2}}{1} \cdot (x-1) + \frac{-\frac{3}{4}}{2} (x-1)^2 = \underline{\frac{3}{2}(x-1) - \frac{3}{8}(x-1)^2}$$

$$R_3(x) = \frac{f'''(\xi)}{3!} \cdot (x - x_0)^3 = \frac{\frac{15}{8\sqrt{\xi^7}}}{6} \cdot (x-1)^3$$

$\longrightarrow \xi$ leží mezi x_0 a x $\quad (x_0 = 1 ; x = \frac{4}{3})$

$$\left|R_3\left(\tfrac{4}{3}\right)\right| \leq \left| \frac{\frac{15}{8\sqrt{1^7}}}{6} \cdot \left(\tfrac{4}{3} - 1\right)^3 \right| = \left| \frac{15}{48} \cdot \frac{1}{27} \right| = \frac{15}{1296} = \frac{5}{432}$$

$$\rightarrow \underline{\underline{\left|R_3\left(\tfrac{4}{3}\right)\right| \leq \frac{5}{432}}}$$

TE1 – jednoduché vzorové otázky

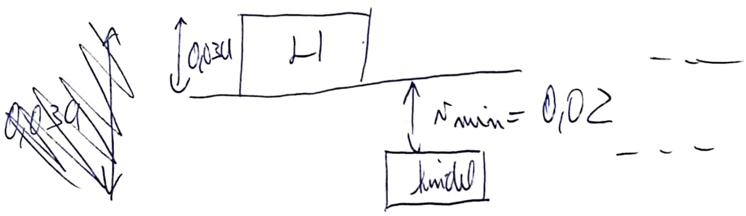
TE2 – ——— " ———

NoM – ——— " ———

ČMS – počítání

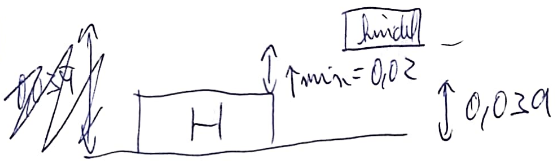
21. -> jednoduchá díra ; šířka t. pole = 0,039 mm

a) -> horní úchylka hřídele -> vůle $r_{min} = 0{,}02$ mm



=> $\underline{\underline{-0{,}02 \text{ mm}}}$

b)



-> $\underline{\underline{+0{,}059 \text{ mm}}}$

22. -> $m = 1$ mm ; $z_1 = 21$; $i = 3{,}1$; $x_1 = 0{,}1$; $x_2 = -0{,}1$

a) $z_2 = z_1 \cdot i \doteq \underline{\underline{65}}$

b) $d_1 = m \cdot z_1 = \underline{\underline{21 \text{ mm}}}$ $\qquad d_2 = m \cdot z_2 = \underline{65 \text{ mm}}$

c) $d_{a1} = d_1 + 2m \cdot (h_a^* + x_1) = 21 + 2 \cdot 1 \cdot (1 + 0{,}1) = \underline{\underline{23{,}2 \text{ mm}}}$

d) $a = \frac{d_1 + d_2}{2} = \frac{65 + 21}{2} = \underline{\underline{43 \text{ mm}}}$

23. přelín → $b = 10\ mm$, $h = 8\ mm$, $l = 40\ mm$

$d = 36\ mm$

$M_k = 180\ Nm$

a) $S_k = \frac{8}{2} \cdot (40 - 10) = \underline{\underline{120\ mm^2}}$

b) $S_\tau = (40 - 10) \cdot 10 = \underline{\underline{300\ mm^2}}$

c) $F = \frac{180000}{18} = \underline{\underline{10\,000\ N}}$

d) $\tau = \frac{F}{S_\tau} = \frac{10\,000}{300} = \underline{\underline{33{,}3\ MPa}}$

PP D: $l = 300\ mm$, $G = 0{,}8 \cdot 10^5$, $\tau_K = 80\ N/mm^2$, $k_k = 2$, $b = 200\ mm$

$h = 3\ mm$

a) $W_K = \frac{1}{3} b \cdot h^2$

b) $M_{KD} = ?\ [Nmm]$

$\tau_K = \frac{M_K}{W_K} \rightarrow M_K = \tau_K \cdot W_k = \tau_K \cdot \frac{1}{3} b \cdot h^2 = 80 \cdot \frac{1}{3} \cdot 200 \cdot 3^2 = 48000$

$\rightarrow M_{KD} = \frac{M_K}{k} = \frac{48\,000}{2} = \underline{\underline{24\,000\ Nmm}}$

c) $\varphi_{A-B} = \frac{M_{KD} \cdot l}{G \cdot J} = \frac{M_{KD} \cdot l}{G \cdot \frac{1}{3} \cdot b \cdot h^3} = \frac{24000 \cdot 300}{0{,}8 \cdot 10^5 \cdot \frac{1}{3} \cdot 200 \cdot 3^3} = \underline{\underline{0{,}05\ rad}} \approx 2{,}86$