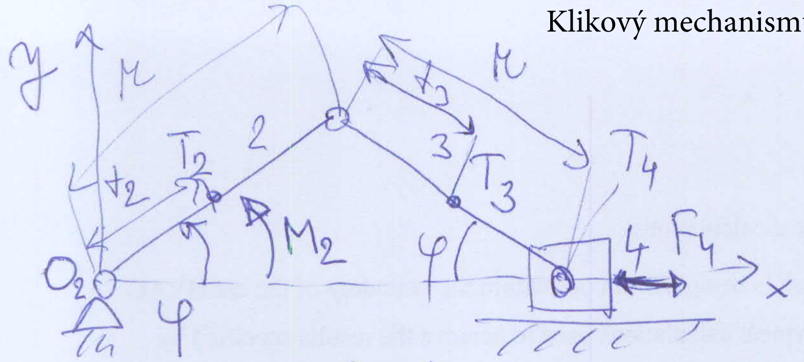
Klikový mechanismus r=l



D: r

$m_2, I_{2T2}, I_{2O2}, t_2$

m_3, I_{3T3}, t_3

m_4, I_{4T4}

na obr. je klikový mechanismus se shodnou délkou kliky a ojnice. Má 1° volnosti. Pomocí souřadnice φ sestavte Lagrangeovými rovnicemi vlastní pohybovou rovnici. Všechny rozměry a hmotové veličiny jsou dány.

1. Napište Lagrangeovy rovnice II druhu a význam symbolů v nich.
2. Sestavte výraz pro kinetickou energii.
3. Určete zobecněnou sílu.
4. Sestavte Lagrangeovy rovnice.

Σ10

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1. $\frac{d}{dt}\frac{\partial E_k}{\partial \dot q} - \frac{\partial E_k}{\partial q} = Q$

E_k - kin. energie
q - zobecněná souřadnice
Q - zobecněná síla

1

2. $E_k = E_{k2} + E_{k3} + E_{k4} =$

$= \frac{1}{2} I_{2O2}\dot\varphi^2 + \frac{1}{2} m_3 v_3^2 + \frac{1}{2} I_{3T3}\omega_3^2 + \frac{1}{2} m_4 v_4^2 + \frac{1}{2} I_{4T4}\omega_4^2$

$\frac{1}{2} m_2 v_2^2 + \frac{1}{2} I_{2T2}\omega_2^2$

2

$x_3 = r\cos\varphi + t_3\cos\varphi$
$y_3 = r\sin\varphi - t_3\sin\varphi$

$\dot x_3 = -r\sin\varphi\dot\varphi - t_3\sin\varphi\dot\varphi = -(r+t_3)\sin\varphi\dot\varphi$
$\dot y_3 = r\cos\varphi\dot\varphi - t_3\cos\varphi\dot\varphi = (r-t_3)\cos\varphi\dot\varphi$

$v_3^2 = ((r+t_3)^2\sin^2\varphi + (r-t_3)^2\cos^2\varphi)\dot\varphi^2 = \dot x_3^2 + \dot y_3^2$

$\omega_4 = 0$

$\omega_3 = -\dot\varphi$

$x_4 = r\cos\varphi + r\cos\varphi = 2r\cos\varphi$
$y_4 = 0$

$v_4^2 = (-2r\sin\varphi\dot\varphi)^2 = 4r^2\sin^2\varphi\dot\varphi^2$

$$E_K = \frac{1}{2} I_{202}\dot{\varphi}^2 + \frac{1}{2} m_3\left((r+t_3)^2 \sin^2\varphi + (r-t_3)^2\cos^2\varphi\right)\dot{\varphi}^2 +$$
$$+ \frac{1}{2} I_{3T3}\dot{\varphi}^2 + \frac{1}{2} m_4 4 r^2 \sin^2\varphi\,\dot{\varphi}^2 \qquad 2$$

3. $$Q\delta q = M_2\delta\varphi + F_4\delta x_4 \qquad 1$$

M_2 na φ $\qquad$ $q = \varphi$

F_4 na x_4

$$Q\delta\varphi = M_2\delta\varphi + F_4\delta(2r\cos\varphi) = M_2\delta\varphi - F_4\cdot 2r\sin\varphi\,\delta\varphi$$
$$Q = M_2 - F_4\cdot 2r\sin\varphi \qquad 1$$

4. $$\frac{\partial E_K}{\partial \dot{q}} = \frac{\partial E_K}{\partial \dot{\varphi}} = I_{202}\dot{\varphi} + m_3\left((r+t_3)^2\sin^2\varphi + (r-t_3)^2\cos^2\varphi\right)\dot{\varphi}$$
$$+ I_{3T3}\dot{\varphi} + m_4 4r^2\sin^2\varphi\,\dot{\varphi}$$

$$\frac{\partial E_K}{\partial q} = \frac{\partial E_K}{\partial \varphi} = m_3\left((r+t_3)^2\sin\varphi\cos\varphi - (r-t_3)^2\cos\varphi\sin\varphi\right)\dot{\varphi}^2 +$$
$$+ m_4\cdot 4r^2\sin\varphi\cos\varphi\,\dot{\varphi}^2$$

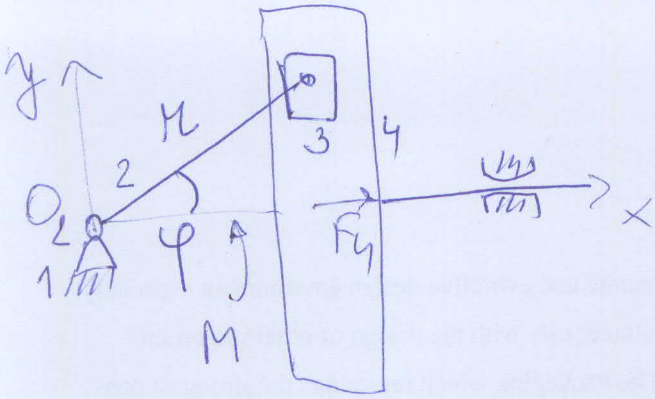
$$\frac{d}{dt}\frac{\partial E_K}{\partial \dot{q}} = \frac{\partial}{\partial \dot{q}}\left(\frac{\partial E_K}{\partial \dot{q}}\right)\ddot{q} + \frac{\partial}{\partial q}\left(\frac{\partial E_K}{\partial \dot{q}}\right)\dot{q} =$$
$$= \frac{\partial}{\partial \dot{\varphi}}\left(\frac{\partial E_K}{\partial \dot{\varphi}}\right)\ddot{\varphi} + \frac{\partial}{\partial \varphi}\left(\frac{\partial E_K}{\partial \varphi}\right)\dot{\varphi} = \qquad 2$$

$$= I_{202}\ddot{\varphi} + m_3\left((r+t_3)^2\sin^2\varphi + (r-t_3)^2\cos^2\varphi\right)\ddot{\varphi} +$$
$$+ I_{3T3}\ddot{\varphi} + m_4\cdot 4r^2\sin^2\varphi\,\ddot{\varphi} + 2m_3\left((r+t_3)^2\sin\varphi\cos\varphi + (r-t_3)^2\right.$$
$$\left.\cdot\cos\varphi(-\sin\varphi)\right)\dot{\varphi}^2 + 2m_4\cdot 4r^2\sin\varphi\cos\varphi\,\dot{\varphi}^2$$

2

$$\left(I_{202} + m_3\left((r+t_3)^2\sin^2\varphi + (r-t_3)\cos^2\varphi\right) + I_{3T3} + m_4\cdot 4r^2\sin^2\varphi\right)\ddot{\varphi} +$$
$$+ m_3\left((r+t_3)^2 - (r-t_3)^2\right)\sin\varphi\cos\varphi\,\dot{\varphi}^2 + m_4\cdot 4r^2\sin\varphi\cos\varphi\,\dot{\varphi}^2 = M_2 - F_4\cdot 2r\sin$$

Kulisový mechanismus



$$E_K = E_{K2} + E_{K3} + E_{K4} = \frac{1}{2} I_{2O_2}\dot\varphi^2 + \frac{1}{2} m_3 v_3^2 + \frac{1}{2} m_4 v_4^2$$

$$x_3 = r\cos\varphi$$
$$y_3 = r\sin\varphi$$

$$\dot x_3 = -r\sin\varphi\dot\varphi \qquad v_3^2 = r^2\dot\varphi^2$$
$$\dot y_3 = r\cos\varphi\dot\varphi$$

$$x_4 = r\cos\varphi$$
$$y_4 = 0$$

$$\dot x_4 = -r\sin\varphi\dot\varphi \qquad v_4^2 = r^2\sin^2\varphi\dot\varphi^2$$
$$\dot y_4 = 0$$

$$E_K = \frac{1}{2} I_{2O2}\dot\varphi^2 + \frac{1}{2} m_3 r^2\dot\varphi^2 + \frac{1}{2} m_4 r^2\sin^2\varphi\dot\varphi^2$$

$$Q\delta q = M\delta\varphi + F_4\delta(x_4)$$

$$Q = M - F_4 r\sin\varphi$$

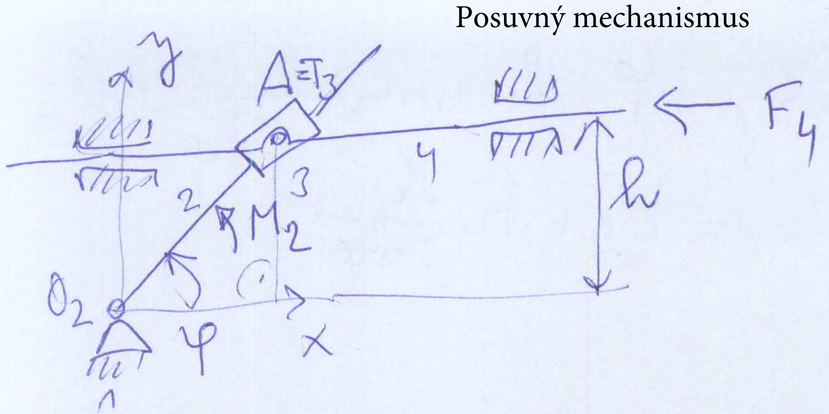
$$\frac{\partial E_K}{\partial\dot\varphi} = \left(I_{2O2} + m_3 r^2 + m_4 r^2\sin^2\varphi\right)\dot\varphi$$

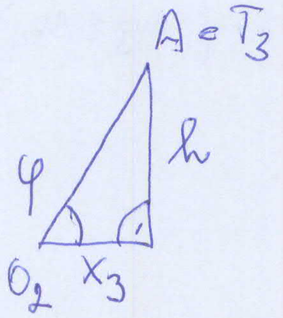
$$\frac{\partial E_K}{\partial\varphi} = m_4 r^2\sin\varphi\cos\varphi\dot\varphi^2$$

$$\frac{d}{dt}\left(\frac{\partial E}{\partial\dot\varphi}\right) = \frac{\partial}{\partial\dot\varphi}\left(\frac{\partial E_K}{\partial\dot\varphi}\right)\ddot\varphi + \frac{\partial}{\partial\varphi}\left(\frac{\partial E_K}{\partial\dot\varphi}\right)\dot\varphi = \left(I_{2O2} + m_3 r^2 + m_4 r^2\sin^2\varphi\right)\ddot\varphi + 2 m_4 r^2\sin\varphi\cos\varphi\dot\varphi^2$$

$$\left(I_{2O2} + m_3 r^2 + m_4 r^2\sin^2\varphi\right)\ddot\varphi + m_4 r^2\sin\varphi\cos\varphi\dot\varphi^2 = M - F_4 r\sin\varphi$$

Posuvný mechanismus





$$E_K = \frac{1}{2} I_{2O_2} \dot{\varphi}^2 + \frac{1}{2} m_3 v_3^2 + \frac{1}{2} I_{3T_3} \omega_3^2 + \frac{1}{2} m_4 v_4^2$$

$$x_3 = \frac{h}{\operatorname{tg}\varphi} \qquad \dot{x}_3 = \left(\frac{h\cos\varphi}{\sin\varphi}\right)^{\cdot} = \frac{-h\sin^2\varphi\,\dot{\varphi} - h\cos^2\varphi\,\dot{\varphi}}{\sin^2\varphi} =$$

$$y_3 = h \qquad \dot{y}_3 = 0 \qquad = -\frac{h}{\sin^2\varphi}\dot{\varphi} \qquad v_3^2 = \frac{h^2\dot{\varphi}^2}{\sin^4\varphi}$$

$$\omega_3 = \dot{\varphi}$$

$$x_4 = \frac{h}{\operatorname{tg}\varphi} \qquad \dot{x}_4 = \frac{-h}{\sin^2\varphi}\dot{\varphi} \qquad v_4^2 = \frac{h^2\dot{\varphi}^2}{\sin^4\varphi}$$

$$y_4 = h \qquad \dot{y}_4 = 0$$

$$E_K = \frac{1}{2} I_{2O_2} \dot{\varphi}^2 + \frac{1}{2} m_3 \frac{h^2\dot{\varphi}^2}{\sin^4\varphi} + \frac{1}{2} I_{3T_3} \dot{\varphi}^2 + \frac{1}{2} m_4 \frac{h^2\dot{\varphi}^2}{\sin^4\varphi}$$

$$Q\delta\varphi = M_2\delta\varphi - F_4\delta x_4 = M_2\delta\varphi - F_4 \frac{-h}{\sin^2\varphi}\delta\varphi$$

$$Q = M_2 + F_4 \frac{h}{\sin^2\varphi}$$

$$\frac{\partial E_K}{\partial \dot{\varphi}} = \left(I_{2O2} + (m_3 + m_4)\frac{h^2}{\sin^4\varphi} + I_{3T3}\right)\dot{\varphi}$$

$$\frac{\partial E_K}{\partial \varphi} = (m_3 + m_4)\frac{h^2}{2}(-4)\sin^{-5}\varphi\cos\varphi\,\dot{\varphi}^2$$

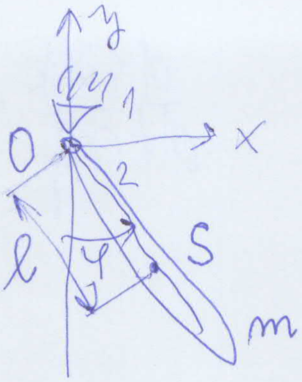
$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{\varphi}}\right) = \frac{\partial}{\partial \dot{\varphi}}\left(\frac{\partial E_K}{\partial \dot{\varphi}}\right)\ddot{\varphi} + \frac{\partial}{\partial \varphi}\left(\frac{\partial E_K}{\partial \varphi}\right)\dot{\varphi} = \left(I_{2O2} + (m_3 + m_4)\frac{h^2}{\sin^4\varphi} + I_{3T3}\right)\ddot{\varphi} +$$

$$+ (m_3 + m_4)\, h^2 (-4) \sin^{-5}\varphi\cos\varphi\,\dot{\varphi}^2$$

$$\left(I_{2O2}+I_{3T3}+(m_3+m_4)\frac{h^2}{\sin^4\varphi}\right)\ddot{\varphi}-2(m_3+m_4)\frac{h^2\cos\varphi}{\sin^5\varphi}\dot{\varphi}^2=$$
$$=M_2+F_4\frac{h}{\sin^2\varphi}$$

Fyzikální kyvadlo

Rři



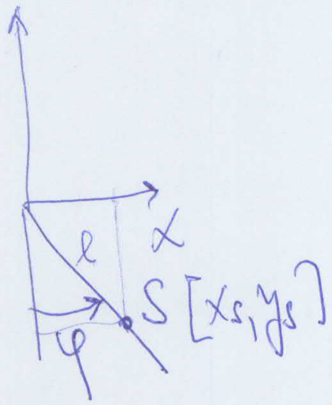
I_s $\quad\downarrow g$

1) $n = 3(2-1) - 2\cdot 1_R = 3-2 = \underline{\underline{1^0}}$

2) $q = \varphi$

3) $E_K = \frac{1}{2} m v^2 + \frac{1}{2} I_s \omega^2$

4)



$x_s = l\sin\varphi \quad \dot{x}_s = l\cos\varphi\,\dot{\varphi} \quad v^2 = \dot{x}_s^2 + \dot{y}_s^2 =$

$y_s = -l\cos\varphi \quad \dot{y}_s = l\sin\varphi\,\dot{\varphi} \quad = l^2(\cos^2\varphi + \sin^2\varphi)\dot{\varphi}^2 =$

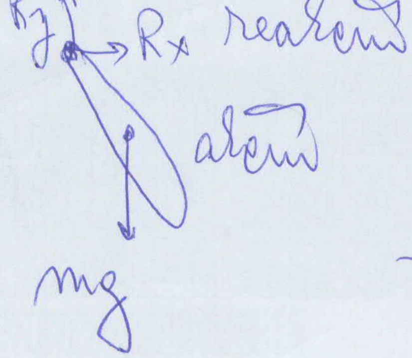
$= l^2\dot{\varphi}^2$

$\omega = \dot{\varphi}$

$E_K = \frac{1}{2} m l^2 \dot{\varphi}^2 + \frac{1}{2} I_s \dot{\varphi}^2 = \frac{1}{2}(ml^2 + I_s)\dot{\varphi}^2 = \underline{\frac{1}{2} I_0 \dot{\varphi}^2}$

Steinerova věta

5) $Q_\varphi \delta\varphi = \begin{bmatrix} 0 \\ -mg \end{bmatrix}^T \begin{bmatrix} \delta x_s \\ \delta y_s \end{bmatrix}$



$\underline{F}^T \delta \underline{r}$

$Q_\varphi \delta\varphi = -mg\,\delta y_s$

6) $y_s = -l\cos\varphi \quad \delta y_s = l\sin\varphi\,\delta\varphi$

$Q_\varphi \delta\varphi = -mg\,l\sin\varphi\,\delta\varphi$

$Q_\varphi = -mgl\sin\varphi$

7) $\frac{d}{dt}\frac{\partial E_K}{\partial \dot{q}} - \frac{\partial E_K}{\partial q} = Q$

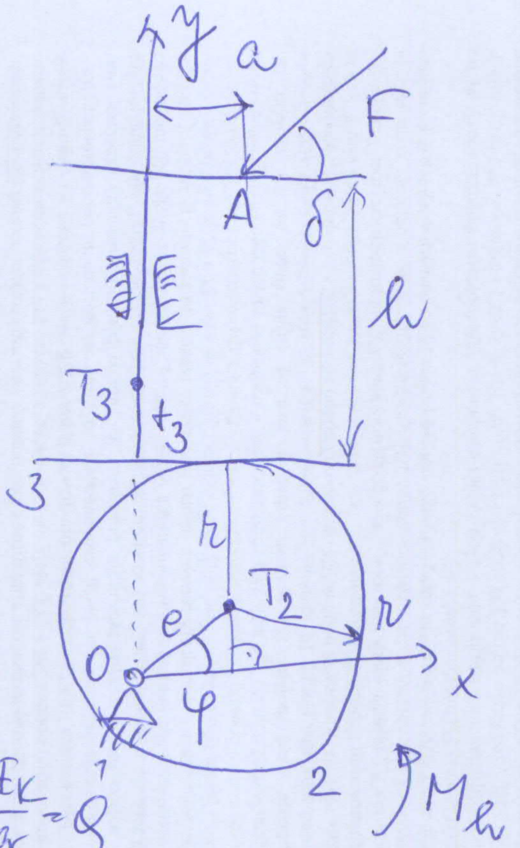
$$\frac{\partial E_k}{\partial \dot{q}} = \frac{1}{2} I_0 \cdot 2\dot{\varphi} = I_0 \dot{\varphi} \qquad \frac{\partial E_k}{\partial q} = 0 \qquad \frac{d}{dt}\frac{\partial E_k}{\partial \dot{q}} = I_0 \ddot{\varphi}$$

$$I_0 \ddot{\varphi} = -mgl \sin\varphi$$

$$\ddot{\varphi} = -\frac{mgl}{I_0} \sin\varphi$$

$$\boxed{\ddot{\varphi} + \frac{mgl}{I_0} \sin\varphi = 0}$$

Vačkový mechanismus



$D: r, e, a, \delta, h$

m_2, I_{2T2}, I_{2O}

m_3, I_{3T3}, t_3

1) $\frac{d}{dt}\frac{\partial E_K}{\partial \dot{q}} - \frac{\partial E_K}{\partial q} = Q$

2) $E_K = \frac{1}{2} m_2 v_2^2 + \frac{1}{2} I_{2T2}\omega_2^2 + \frac{1}{2} m_3 v_3^2 + \frac{1}{2} I_{3T3}\underbrace{\omega_3^2}_{0} =$

$= \frac{1}{2} I_{2O}\omega_2^2 + \frac{1}{2} m_3 v_3^2$

$\omega_2 = \dot{\varphi}$

$x_3 = 0 \qquad \dot{x}_3 = 0 \qquad v_3^2 = \dot{x}_3^2 + \dot{y}_3^2 = e^2 c^2\varphi\,\dot{\varphi}^2$

$y_3 = e\sin\varphi + r + t_3 \qquad \dot{y}_3 = e\cos\varphi\,\dot{\varphi}$

$E_K = \frac{1}{2} I_{2O}\dot{\varphi}^2 + \frac{1}{2} m_3 e^2 c^2\varphi\,\dot{\varphi}^2$

3) $Q\delta q = M_h\delta\varphi - F\sin\delta\ \delta(y_A)$

$y_A = e\sin\varphi + r + h \qquad \delta y_A = e c\varphi\,\delta\varphi$

$Q = M_h - F s\delta\, e c\varphi$

4) $$\frac{\partial E_K}{\partial \dot q} = \frac{\partial E_K}{\partial \dot\varphi} = \left(I_{20} + m_3 e^2 c^2\varphi\right)\dot\varphi$$

$$\frac{\partial E_K}{\partial q} = \frac{\partial E_K}{\partial \varphi} = \frac{1}{2} m_3 e^2\, 2c\varphi(-s\varphi)\,\dot\varphi^2 = -m_3 e^2 c\varphi s\varphi \dot\varphi^2$$

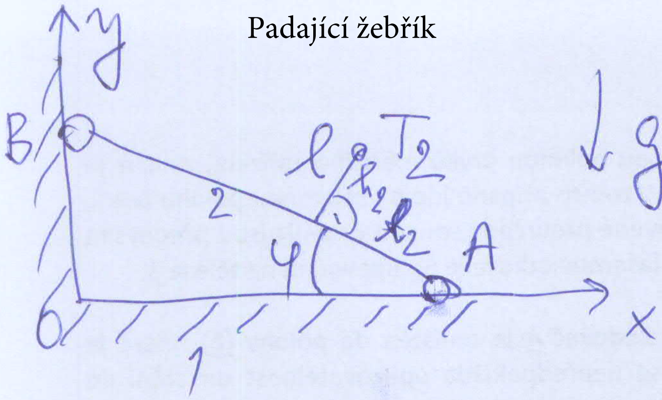
$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot q}\right) = \frac{\partial}{\partial \dot q}\left(\frac{\partial E_K}{\partial \dot q}\right)\ddot q + \frac{\partial}{\partial q}\left(\frac{\partial E_K}{\partial \dot q}\right)\dot q =$$

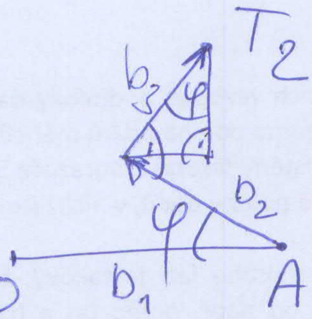
$$= \frac{\partial}{\partial \dot\varphi}\left(\frac{\partial E_K}{\partial \dot\varphi}\right)\ddot\varphi + \frac{\partial}{\partial \varphi}\left(\frac{\partial E_K}{\partial \dot\varphi}\right)\dot\varphi =$$

$$= \left(I_{20} + m_3 e^2 c^2\varphi\right)\ddot\varphi + m_3 e^2 \cdot 2c\varphi(-s\varphi)\dot\varphi^2$$

$$\left(I_{20} + m_3 e^2 c^2\varphi\right)\ddot\varphi - m_3 e^2 c\varphi s\varphi \dot\varphi^2 = M_{el} - F_0 e c\varphi$$

Padající žebřík





$D: l, l_2, h_2, m_2, I_{2T2}$

2) $E_K = \frac{1}{2} m_2 v_2^2 + \frac{1}{2} I_{2T2}\, \omega_2^2 =$

$x_A = l c\varphi$

$$\begin{bmatrix} x_{T2} \\ y_{T2} \end{bmatrix} = \begin{bmatrix} l c\varphi \\ 0 \end{bmatrix} + \begin{bmatrix} -l_2 c\varphi \\ l_2 s\varphi \end{bmatrix} + \begin{bmatrix} h_2 s\varphi \\ h_2 c\varphi \end{bmatrix}$$

$\underline{r}_{T2} = \underline{b}_1 + \underline{b}_2 + \underline{b}_3$

$x_{T2} = (l - l_2) c\varphi + h_2 s\varphi \qquad \dot{x}_{T2} = \left((l - l_2)(-s\varphi) + h_2 c\varphi\right)\dot{\varphi}$

$y_{T2} = l_2 s\varphi + h_2 c\varphi \qquad \dot{y}_{T2} = l_2 c\varphi\, \dot{\varphi} + h_2(-s\varphi)\dot{\varphi}$

$v_2^2 = \dot{x}_{T2}^2 + \dot{y}_{T2}^2 = \left(\left((l - l_2) s\varphi - h_2 c\varphi\right)^2 + \left(l_2 c\varphi - h_2 s\varphi\right)^2\right)\dot{\varphi}^2$

$\omega_2 = -\dot{\varphi}$

$E_K = \frac{1}{2} m_2 \left(\left((l - l_2) s\varphi - h_2 c\varphi\right)^2 + \left(l_2 c\varphi - h_2 s\varphi\right)^2\right)\dot{\varphi}^2 + \frac{1}{2} I_{2T2}\dot{\varphi}^2$

3) $Q\,\delta q = -m_2 g\, \delta(y_{T2})$

$Q\,\delta\varphi = -m_2 g (l_2 c\varphi - h_2 s\varphi)\,\delta\varphi$

$Q = -m_2 g (l_2 c\varphi - h_2 s\varphi)$

4) $$\frac{\partial E_K}{\partial \dot{q}} = \frac{\partial E_K}{\partial \dot{\varphi}} = m_2\left(\left((l-l_2)S\varphi - h_2C\varphi\right)^2 + \left(l_2C\varphi - h_2S\varphi\right)^2\right)\dot{\varphi} + I_{2T2}\dot{\varphi}$$

$$\frac{\partial E_K}{\partial q} = \frac{\partial E_K}{\partial \varphi} = m_2\Big(\left((l-l_2)S\varphi - h_2C\varphi\right)\left((l-l_2)C\varphi + h_2S\varphi\right) + \left(l_2C\varphi - h_2S\varphi\right)\left(-l_2S\varphi - h_2C\varphi\right)\Big)\dot{\varphi}^2$$

$$\frac{d}{dt}\frac{\partial E_K}{\partial \dot{q}} = \frac{\partial}{\partial \dot{q}}\left(\frac{\partial E_K}{\partial \dot{q}}\right)\ddot{q} + \frac{\partial}{\partial q}\left(\frac{\partial E_K}{\partial \dot{q}}\right)\dot{q} = \frac{\partial}{\partial \dot{\varphi}}\left(\frac{\partial E_K}{\partial \dot{\varphi}}\right)\ddot{\varphi} + \frac{\partial}{\partial \varphi}\left(\frac{\partial E_K}{\partial \dot{\varphi}}\right)\dot{\varphi} =$$

$$= \left(m_2\left(\left((l-l_2)S\varphi - h_2C\varphi\right)^2 + \left(l_2C\varphi - h_2S\varphi\right)^2\right) + I_{2T2}\right)\ddot{\varphi} +$$

$$+ 2m_2\Big(\left((l-l_2)S\varphi - h_2C\varphi\right)\left((l-l_2)C\varphi + h_2S\varphi\right) + \left(l_2C\varphi - h_2S\varphi\right)\left(-l_2S\varphi - h_2C\varphi\right)\Big)\dot{\varphi}^2$$

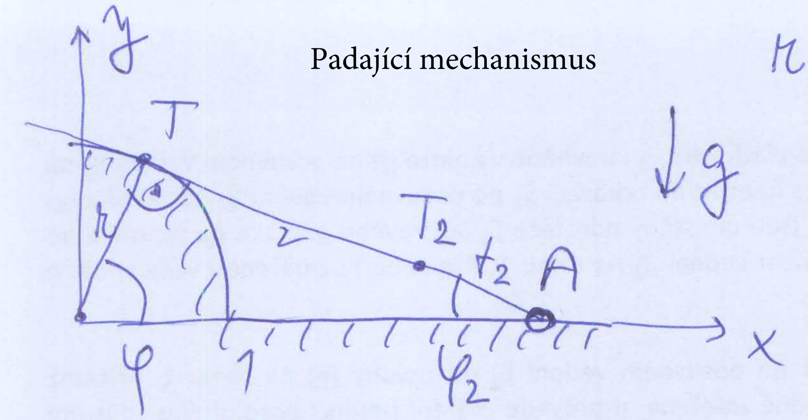
$$\left(m_2\left(\left((l-l_2)S\varphi - h_2C\varphi\right)^2 + \left(l_2C\varphi - h_2S\varphi\right)^2\right) + I_{2T2}\right)\ddot{\varphi} +$$

$$+ m_2\Big(\left((l-l_2)S\varphi - h_2C\varphi\right)\left((l-l_2)C\varphi + h_2S\varphi\right) + \left(l_2C\varphi - h_2S\varphi\right)\left(-l_2S\varphi - h_2C\varphi\right)\Big)\dot{\varphi}^2 =$$

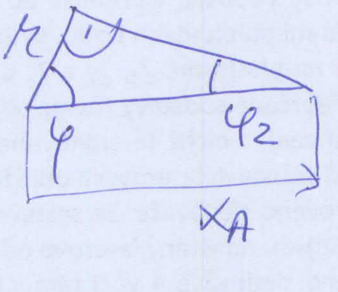
$$= -m_2g\left(l_2C\varphi - h_2S\varphi\right)$$

Padající mechanismus

r, t_2, m_2, I_{2T2}



$$E_K = E_{K2} = \frac{1}{2} m_2 v_2^2 + \frac{1}{2} I_{2T2}\omega_2^2 =$$



$$x_A c\varphi = r$$

$$x_A = \frac{r}{c\varphi}$$

$$\varphi_2 = \frac{\pi}{2} - \varphi$$

$$x_{T2} = x_A - t_2 c\varphi_2$$

$$y_{T2} = t_2 \sin\varphi_2$$

$$x_{T2} = \frac{r}{c\varphi} - t_2 c\left(\frac{\pi}{2} - \varphi\right) = \frac{r}{c\varphi} - t_2 s\varphi$$

$$y_{T2} = t_2 s\left(\frac{\pi}{2} - \varphi\right) = t_2 c\varphi$$

$$\dot{x}_{T2} = \frac{-r(-s\varphi)}{c^2\varphi}\dot{\varphi} - t_2 c\varphi\dot{\varphi} = +\left(r\frac{s\varphi}{c^2\varphi} - t_2 c\varphi\right)\dot{\varphi}$$

$$\dot{y}_{T2} = -t_2 s\varphi\dot{\varphi}$$

$$v_2^2 = \left(\left(r\frac{s\varphi}{c^2\varphi} - t_2 c\varphi\right)^2 + (t_2 s\varphi)^2\right)\dot{\varphi}^2$$

$$\omega_2 = -\dot{\varphi}_2 = +\dot{\varphi}$$

$$E_K = \frac{1}{2} m_2 \left(\left(r\frac{s\varphi}{c^2\varphi} - t_2 c\varphi\right)^2 + (t_2 s\varphi)^2\right)\dot{\varphi}^2 + \frac{1}{2} I_{2T2}\dot{\varphi}^2$$

3) $$Q\,\delta q = -m_2 g\,\delta y_{T2}$$

$$Q\,\delta\varphi = -m_2 g\,(-t_2 s\varphi)\,\delta\varphi$$

$$Q = m_2 g\, t_2 s\varphi$$

4) $$\frac{\partial E_K}{\partial \dot q} = \frac{\partial E_K}{\partial \dot\varphi} = m_2\left(\left(r\frac{s\varphi}{c^2\varphi} - t_2 c\varphi\right)^2 + \left(t_2 s\varphi\right)^2\right)\dot\varphi + I_{2T2}\dot\varphi$$

$$\frac{\partial E_K}{\partial q} = \frac{\partial E_K}{\partial \varphi} = m_2\left(\left(r\frac{s\varphi}{c^2\varphi} - t_2 c\varphi\right)\left(r\frac{c\varphi c^2\varphi - c\varphi\cdot 2c\varphi(-s\varphi)}{c^4\varphi} + t_2 s\varphi\right)\right)\dot\varphi^2$$

$t_2^2 s\varphi c\varphi$

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot q}\right) = \frac{\partial}{\partial \dot q}\left(\frac{\partial E_K}{\partial \dot q}\right)\ddot q + \frac{\partial}{\partial q}\left(\frac{\partial E_K}{\partial \dot q}\right)\dot q =$$

$$= \frac{\partial}{\partial \dot\varphi}\left(\frac{\partial E_K}{\partial \dot\varphi}\right)\ddot\varphi + \frac{\partial}{\partial \varphi}\left(\frac{\partial E_K}{\partial \dot\varphi}\right)\dot\varphi =$$

$$= \left(m_2\left(\left(r\frac{s\varphi}{c^2\varphi} - t_2 c\varphi\right)^2 + \left(t_2 s\varphi\right)^2\right) + I_{2T2}\right)\ddot\varphi +$$

$$+ 2m_2\left(\left(r\frac{s\varphi}{c^2\varphi} - t_2 c\varphi\right)\left(r\frac{c\varphi c^2\varphi - c\varphi\cdot 2c\varphi(-s\varphi)}{c^4\varphi} + t_2 s\varphi\right)\right)\dot\varphi^2$$

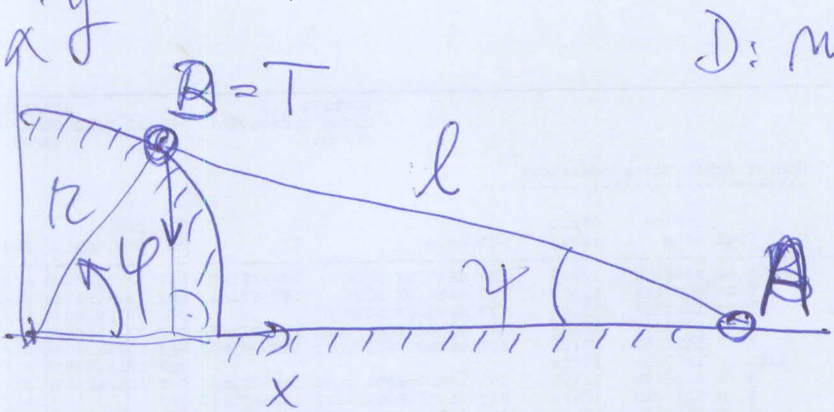
$t_2^2 s\varphi c\varphi$

$$\left(m_2\left(\left(r\frac{s\varphi}{c^2\varphi} - t_2 c\varphi\right)^2 + \left(t_2 s\varphi\right)^2\right) + I_{2T2}\right)\ddot\varphi +$$

$$+ m_2\left(\left(r\frac{s\varphi}{c^2\varphi} - t_2 c\varphi\right)\left(r\frac{c\varphi + 2s\varphi}{c^2\varphi} + t_2 s\varphi\right) + t_2^2 s\varphi c\varphi\right)\dot\varphi^2 =$$

$$= m_2 g\, t_2 s\varphi$$

Padající žebřík II



$D: m, I_T, r, l$

2) $E_K = \frac{1}{2} m v^2 + \frac{1}{2} I_T \omega^2$

$x_T = r c\varphi \qquad \dot{x}_T = -r s\varphi \dot{\varphi} \qquad v^2 = r^2 \dot{\varphi}^2$

$y_T = r s\varphi \qquad \dot{y}_T = r c\varphi \dot{\varphi}$

$\omega = \dot{\psi}$

$r s\varphi = l \sin\psi$

$r c\varphi \dot{\varphi} = l c\psi \dot{\psi}$

$\dot{\psi} = \frac{r}{l} \frac{c\varphi}{c\psi} \dot{\varphi}$

$c\psi = \sqrt{1 - s^2\psi} = \sqrt{1 - \left(\frac{r}{l} s\varphi\right)^2}$

$\dot{\psi} = \frac{r}{l} \frac{c\varphi}{\sqrt{1 - \left(\frac{r}{l}\right)^2 s^2\varphi}} \dot{\varphi}$

$E_K = \frac{1}{2} m r^2 \dot{\varphi}^2 + \frac{1}{2} I_T \left(\frac{r}{l} \frac{c\varphi}{\sqrt{1 - \left(\frac{r}{l} s\varphi\right)^2}}\right)^2 \dot{\varphi}^2$

3) $Q\delta q = -mg\,\delta y_T = -mg\,r c\varphi\,\delta\varphi$

$$\underline{Q = -mg\,r c\varphi}$$

4) $$\frac{\partial E_K}{\partial \dot q} = \frac{\partial E_K}{\partial \dot\varphi} = \left(mr^2 + I_T\left(\frac{r}{l}\right)^2 \frac{c^2\varphi}{1-\left(\frac{r}{l}s\varphi\right)^2}\right)\dot\varphi$$

$$\frac{\partial E_K}{\partial q} = \frac{\partial E_K}{\partial \varphi} = \frac{1}{2} I_T\left(\frac{r}{l}\right)^2 \frac{2c\varphi(-s\varphi)\left(1-\left(\frac{r}{l}s\varphi\right)^2\right) - c^2\varphi\left(-\left(\frac{r}{l}\right)^2 2s\varphi c\varphi\right)}{\left(1-\left(\frac{r}{l}s\varphi\right)^2\right)^2}\dot\varphi^2 =$$

$$= I_T\left(\frac{r}{l}\right)^2 \frac{(-s\varphi c\varphi)\left(1-\left(\frac{r}{l}s\varphi\right)^2 - \left(\frac{r}{l}\right)^2 c^2\varphi\right)}{\left(1-\left(\frac{r}{l}s\varphi\right)^2\right)^2}\dot\varphi^2 =$$

$$= -I_T\left(\frac{r}{l}\right)^2 \frac{s\varphi c\varphi\left(1-\left(\frac{r}{l}\right)^2\right)}{\left(1-\left(\frac{r}{l}s\varphi\right)^2\right)^2}\dot\varphi^2$$

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot q}\right) = \frac{\partial}{\partial \dot q}\left(\frac{\partial E_K}{\partial \dot q}\right)\ddot q + \frac{\partial}{\partial q}\left(\frac{\partial E_K}{\partial \dot q}\right)\dot q = \frac{\partial}{\partial \dot\varphi}\left(\frac{\partial E_K}{\partial \dot\varphi}\right)\ddot\varphi + \frac{\partial}{\partial \varphi}\left(\frac{\partial E_K}{\partial \dot\varphi}\right)\dot\varphi =$$

$$= \left(mr^2 + I_T\left(\frac{r}{l}\right)^2 \frac{c^2\varphi}{1-\left(\frac{r}{l}s\varphi\right)^2}\right)\ddot\varphi - I_T\left(\frac{r}{l}\right)^2 \frac{2s\varphi c\varphi\left(1-\left(\frac{r}{l}\right)^2\right)}{\left(1-\left(\frac{r}{l}s\varphi\right)^2\right)^2}\dot\varphi^2$$

$$\left(mr^2 + I_T\left(\frac{r}{l}\right)^2 \frac{c^2\varphi}{1-\left(\frac{r}{l}s\varphi\right)^2}\right)\ddot\varphi - I_T\left(\frac{r}{l}\right)^2 \frac{s\varphi c\varphi\left(1-\left(\frac{r}{l}\right)^2\right)}{\left(1-\left(\frac{r}{l}s\varphi\right)^2\right)^2}\dot\varphi^2 =$$

$$= -mg\,r c\varphi$$